Decoupling limits in Renormalizable Quantum Gravity

Luca Buoninfante

Work in progress…[arXiv:2305.XXXXX]

Talk @ Quantum Gravity & Cosmology
19th April 2023

NORDITA
The Nordic Institute for Theoretical Physics

Some remarks on Renormalizable Quantum Gravity

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Renormalizable Quantum Gravity

The following extension of Einstein's General Relativity (GR) is 'strictly' renormalizable (in $D = 4$): [Stelle, PRD (1977)]

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\left(\gamma(R - 2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right), \qquad M_p^2 = \frac{\gamma}{\kappa^2}$$

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Usual story: assume $\Lambda \simeq 0$, expand $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$, compute the propagator

$$\Pi(k) = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}^{(0)}}{k^2 + m_0^2} - \frac{\mathcal{P}^{(2)}}{k^2 + m_2^2}, \qquad m_0^2 = \frac{\gamma}{\alpha}, \qquad m_2^2 = \frac{\gamma}{\beta}$$

Spin-2 massive ghost

Reason why such a QFT of gravity is usually claimed to be pathological when quantized with conventional methods!

Outline of the talk

First part (*reasons why I would understand the theory better*)

1. Motivations for strictly renormalizable quantum gravity: a different way to tell the story;
2. Brief review of some peculiar features of strictly renormalizable quantum gravity.

Second part (*some aspects I find very interesting!*)

1. Questions: what are the limits of the theory when

$$\alpha \to \infty \quad \text{or} \quad \beta \to \infty \quad ?$$

2. A non-zero cosmological constant affects the limits, in particular $\beta \to \infty$
3. Different meanings for $\beta \to \infty$ when $\Lambda = 0$ or $\Lambda \neq 0$
4. Renormalizability avoids strong coupling in the limit $\beta \to \infty$
5. High-energy limit of the theory and the role of $\Lambda \neq 0$
6. Work in progress...and discussions...

Motivations

The GR Lagrangian alone $S_{GR} = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R$

cannot be used to describe at least two physical observed phenomena:

1. Current accelerated expansion of the Universe (*late time physics*);
2. CMB anisotropies (*early time physics*);

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Simplest phenomenological model to explain observations:

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}R + S_\Lambda + S_\phi + \cdots,$$

$$S_\Lambda \equiv \frac{1}{\kappa^2}\int d^4x\,\sqrt{-g}\,\Lambda,$$

$S_\phi \equiv$ scalar field (inflaton) with some suitable self-potential

The additional free parameters in S_Λ and S_ϕ can be fixed by matching with observations.

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Question: is there any way to derive those additional terms in the Lagrangian from some ‘guiding principles’?

Motivations

A logical way to proceed:

Framework of QFT and criterion of 'strict' renormalizability to select fundamental theories: very successful to describe electro-weak and strong interactions

Let's apply it to gravity and check whether it can be useful to describe Nature:

- If it fails, then do something else;
- BUT, if it doesn't fail, then try to understand the theory better!

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Let's apply it to gravity and check whether it can be useful to describe Nature:

- If it fails, then do something else;
- BUT, if it doesn't fail, then try to understand the theory you get better!

The criterion of strict renormalizability selects the gravitational action:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(\gamma(R-2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right), \qquad M_p^2 = \frac{\gamma}{\kappa^2}$$

Cosmological constant: $\Lambda \sim 10^{-122} M_p^2$

Additional scalar field: $\frac{\alpha}{\kappa^2} \sim 10^{10}$

Natural explanation for inflation and CMB anisotropies! [Starobinsky, 1980+]

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In my opinion, very important implications follow if we accept these facts:

1. The framework of perturbative QFT and the criterion of strict renormalizability (as a tool to select theories) are quite successful also when applied to gravity!
2. CMB observations have provided for the first time a test of higher-curvature gravity and an 'indirect' proof of quantized gravity (the scalar field is a gravitational dof)!!
3. Contrary to some beliefs, Starobinsky inflation is not just a model!

Motivations

Obvious question: What about the spin-2 massive ghost?

Given the different way I've told the story about strictly renormalizable quantum gravity,

1. Would you throw the entire theory away just because maybe we don't understand or we don't know how to deal with the spin-2 ghost?

2. Or, instead, after appreciating the achievements described before, should you feel very motivated to understand the role of the ghost at a deeper level?

I opt for the 2nd option!

Some interesting features of the theory

- The loop expansion is controlled by the couplings $g_0 \equiv \frac{1}{\alpha}$ and $g_2 \equiv \frac{1}{\beta}$.

- Unlike the EFT approach to Einstein's GR, α and β can be large: the bigger, the better for the perturbative expansion! Reason why Starobinsky inflation can be consistently embedded in a renormalizable theory but <u>not</u> in the EFT approach to Einstein's GR.

- If α and β are positive: g_2 is asymptotically free (i.e. $\beta \to \infty$), whereas g_0 grows logarithmically with the energy (i.e. $\alpha \to 0$).

  [Julve, Tonin (1978); Fradkin, Tseytlin (1982); Avramidi, Barvinsky(1985+); Salvio, Strumia (2014+); Piva (2021)]

- The tree-level graviton-graviton scattering is the same as Einstein's GR, i.e. the cross section still grows as $\sim \frac{E^2}{M_p^2}$. [Donà et al. (2015); Holdom (2021)]
- The last point might be related to the presence of semi-classical (black-hole) states in the spectrum. [But see Holdom (2021) for a different perspective]

Recent proposals to recover unitarity with ghost

S-matrix unitarity and optical theorem:

$$S^{+}S = 1, \qquad S = 1 + iT,$$
$$1 = \sum_{\{n\}} c_n \, |n\rangle\langle n| \qquad \Longrightarrow \qquad 2Im\{\langle a|T|a\rangle\} = \sum_{\{n\}} c_n |\langle n|T|a\rangle|^2$$

Interesting approaches

- Quantize the ghost with negative norms ($c_n < 0$ for ghost states but positive energies) [Salvio, Strumia (2014+); Holdom (2021+); etc]
- Loop corrections make the ghost decay after times of order $\tau \sim M_p^2/m_2^3$: treat the ghost as an unstable particle, unitarity restored for $t > \tau$ [Donoghue, Menezes (2018+)]
- Replace the Feynman $i\epsilon$ with the *Fakeon* prescription and convert the ghost into a purely virtual particle (LHS=0 for ghost cuts and $c_n = 0$ for ghost states) [Anselmi & Piva 2017+]

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Aim of this talk

$$S = \frac{1}{2\kappa^2}\int d^4x\,\sqrt{-g}\left(\gamma(R-2\Lambda)+\frac{\alpha}{6}R^2-\frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right), \qquad M_p^2=\frac{\gamma}{\kappa^2}$$

Questions:

$$\alpha \to \infty \quad \Longrightarrow \quad S \to ?$$

$$\beta \to \infty \quad \Longrightarrow \quad S \to ?$$

To answer these questions, it is convenient to rewrite the action in an equivalent form where all the degrees of freedom, their masses and couplings are explicit.

Additional spin-0 field

1. Auxiliary scalar field χ

$$S[g,\chi] = \frac{\bar{\gamma}}{2\kappa^2}\int d^4x\sqrt{-g}(R-2\Lambda) + S_W[g] + \frac{\alpha}{12\kappa^2}\int d^4x\sqrt{-g}(2R-8\Lambda-\chi)\chi$$

2. Conformal transformation: $g_{\mu\nu} \to \frac{1}{1+\alpha\chi/3\bar{\gamma}} g_{\mu\nu}$

3. Canonical normalization: $\phi = \sqrt{\frac{3\bar{\gamma}}{2\kappa^2}\frac{\alpha}{3\bar{\gamma}}}\chi$

$$S[g,\phi] = \frac{\bar{\gamma}}{2\kappa^2}\int d^4x\sqrt{-g}(R-2\Lambda) + S_W[g] + S_0[g,\phi],$$

$$S_0[g,\phi] = \int d^4x\sqrt{-g}\frac{1}{\left(1+\sqrt{2\kappa^2/3\bar{\gamma}}\phi\right)^2}\left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m_0^2}{2}\phi^2\right)$$

$$\bar{\gamma} \equiv \gamma + \frac{4}{3}\alpha\Lambda$$

Rescaled Planck Mass when $\Lambda \neq 0$

$$m_0^2 \equiv \frac{\gamma}{\alpha}$$

Mass of the scalar field

Additional spin-0 field: couplings

We can expand around $\phi = 0$:

$$S[g,\phi] = \frac{\bar{\gamma}}{2\kappa^2}\int d^4x\sqrt{-g}(R-2\Lambda) + S_W[g] + S_0[g,\phi],$$

$$S_0[g,\phi] = \int d^4x\sqrt{-g}\left(1 - 2\sqrt{\frac{2\kappa^2}{3\bar{\gamma}}}\phi + \cdots + (n-1)(-1)^{n-2}\left(\frac{2\kappa^2}{3\bar{\gamma}}\right)^{\frac{n-2}{2}}\phi^{n-2} + \cdots\right) \times\left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m_0^2}{2}\phi^2\right)$$

Derivative interaction couplings $\sim \left(\frac{\kappa^2}{\bar{\gamma}}\right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p}\right)^{n-2}\left(\frac{1}{1+4\alpha\Lambda/3\gamma}\right)^{\frac{n-2}{2}}$

Non-derivative interaction couplings $\sim m_0^2\left(\frac{\kappa^2}{\bar{\gamma}}\right)^{\frac{n-2}{2}} = \frac{\gamma}{\alpha}\left(\frac{1}{M_p}\right)^{n-2}\left(\frac{1}{1+4\alpha\Lambda/3\gamma}\right)^{\frac{n-2}{2}}$

Interaction couplings depend on Λ !

Additional spin-2 field

1. Auxiliary spin-2 field $\varphi_{\mu\nu}$ [Kaku et al. (1977); Hindawi et al. (1996); Anselmi & Piva (2018)]

$$S[g,\phi,\varphi] = \frac{\tilde{\gamma}}{2\kappa^2}\int d^4x\,\sqrt{-g}(R-2\Lambda) + S_0[g,\phi]$$
$$-\frac{\beta}{2\kappa^2}\int d^4x\,\sqrt{-g}\left[\bar{\gamma}(G_{\mu\nu}+\Lambda g_{\mu\nu})\varphi^{\mu\nu} - \frac{\bar{\gamma}^2}{4}(\varphi_{\mu\nu}\varphi^{\mu\nu}-\varphi^2)\right]$$

2. Metric transformation: $g_{\mu\nu} \to g_{\mu\nu} - \beta\frac{\bar{\gamma}}{\tilde{\gamma}}\varphi_{\mu\nu}$

3. Canonical normalization: $f_{\mu\nu} = \beta\sqrt{\frac{\bar{\gamma}}{4\kappa^2}\frac{\bar{\gamma}}{\tilde{\gamma}}}\varphi_{\mu\nu}$

$$S[g,\phi,f] = \frac{\tilde{\gamma}}{2\kappa^2}\int d^4x\,\sqrt{-g}(R-2\Lambda) + S_0\left[g-\sqrt{2\kappa^2/\tilde{\gamma}}f,\phi\right] + S_2[g,f],$$

$$\boxed{\tilde{\gamma} \equiv \bar{\gamma} + \frac{2}{3}\beta\Lambda = \gamma + \frac{2}{3}(2\alpha+\beta)\Lambda}$$

Additional rescaling of
Planck Mass when $\Lambda \neq 0$

Additional spin-2 field

Action for the massive spin-2 around $f_{\mu\nu} = 0$:

$$S_2[g,f] = -S_{PF}[g,f] - \int d^4x\sqrt{-g}\left[\left(2f_\mu^\rho f_{\rho\nu} - f f_{\mu\nu}\right)R^{\mu\nu} - \frac{R}{2}\left(f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}f^2\right)\right]$$

$$-\frac{1}{2}\sqrt{\frac{\kappa^2}{\tilde{\gamma}}}m_2^2\int d^4x\sqrt{-g}\left[5f_{\mu\nu}f^{\mu\nu}f - 4f^{\mu\nu}f_\mu^\rho f_{\rho\nu} - f^3\right]$$

$$+\frac{8}{3}\frac{\kappa^2}{\gamma}\sqrt{\frac{\kappa^2}{\tilde{\gamma}}}\int d^4x d^4y d^4z\sqrt{-g}\frac{\delta^{(3)}S_{EH}}{\delta g_{\mu\nu}(x)\delta g_{\rho\sigma}(y)\delta g_{\alpha\beta}(z)}f_{\mu\nu}(x)f_{\rho\sigma}(y)f_{\alpha\beta}(z)$$

$$+O(f^4)$$

$S_{PF}[g,f]$ is the Fierz-Pauli action for the field $f_{\mu\nu}$ in the metric $g_{\mu\nu}$, with mass

$$m_2^2 = \frac{\gamma}{\beta} + \frac{2}{3}\left(2\frac{\alpha}{\beta} + 1\right)\Lambda$$

The mass of the spin-2 ghost depends on Λ !

$$m_2^2 \geq \frac{2}{3}\Lambda$$

(If $\Lambda \geq 0,\ m_2^2 \geq 0,\ \beta > 0$)

Additional spin-2 field: couplings

- n-point interaction couplings generated by variations of S_{EH}

$$\sim \left(\frac{\kappa^2}{\tilde{\gamma}}\right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p}\right)^{n-2} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3\gamma}\right)^{\frac{n-2}{2}}$$

- n-point interaction couplings generated by the mass term

$$\sim m_2^2 \left(\frac{\kappa^2}{\tilde{\gamma}}\right)^{\frac{n-2}{2}} = \frac{\gamma}{\beta} \left(\frac{1}{M_p}\right)^{n-2} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3\gamma}\right)^{\frac{n-2}{2}}$$

Interaction couplings depend on Λ $\Longrightarrow$ additional dependences on β !

Couplings for the massless spin-2

When $\Lambda \neq 0$ the couplings of the massless spin-2 also change:

$$\frac{\tilde{\gamma}}{2\kappa^2}\int d^4x\sqrt{-g}(R-2\Lambda) \quad \rightarrow \quad g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{2\kappa}{\sqrt{\tilde{\gamma}}}h_{\mu\nu}, \qquad \tilde{\gamma} = \gamma + \frac{2}{3}(2\alpha+\beta)\Lambda$$

Self-interaction couplings

$$\sim \left(\frac{\kappa^2}{\tilde{\gamma}}\right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p}\right)^{n-2}\left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3\gamma}\right)^{\frac{n-2}{2}}$$

When $\Lambda \neq 0 \quad \Longrightarrow \quad$ additional dependences on Λ, α, β !

Limit $\alpha \to \infty$ with $\Lambda = 0$

For the time being, let us consider the spin-0 action around $\bar{g}_{\mu\nu}$

$$S_0[\bar{g},\phi] = \int d^4x \sqrt{-\bar{g}} \left(1 - 2\sqrt{\frac{2\kappa^2}{3\bar{\gamma}}}\phi + \cdots + (n-1)(-1)^{n-2}\left(\frac{2\kappa^2}{3\bar{\gamma}}\right)^{\frac{n-2}{2}} \phi^{n-2} + \cdots \right) \times \left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m_0^2}{2}\phi^2 \right)$$

When $\Lambda = 0$, the limit $\alpha \to \infty$ (other parameters and fields kept fixed) gives

- Derivative couplings $\sim \left(\frac{\kappa^2}{\gamma}\right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p}\right)^{n-2} \to$ finite
- Non-derivative couplings $\sim m_0^2 \left(\frac{\kappa^2}{\gamma}\right)^{\frac{n-2}{2}} = \frac{\gamma}{\alpha}\left(\frac{1}{M_p}\right)^{n-2} \to 0$

$$\Longrightarrow S_0[\bar{g},\phi] = \int d^4x \sqrt{-\bar{g}} \left(1 - 2\sqrt{\frac{2\kappa^2}{3\bar{\gamma}}}\phi + \cdots + (n-1)(-1)^{n-2}\left(\frac{2\kappa^2}{3\bar{\gamma}}\right)^{\frac{n-2}{2}} \phi^{n-2} + \cdots \right)\left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi \right)$$

$\alpha \to \infty$ is a massless limit & the scalar field is still self-interacting (when $\Lambda = 0$)!

Limit $\alpha \to \infty$ with $\Lambda \neq 0$

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$$\times \left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi - \frac{m_0^2}{2}\phi^2 \right)$$

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- Derivative couplings $\sim \left(\frac{\kappa^2}{\bar{\gamma}}\right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p}\right)^{n-2} \left(\frac{1}{1+4\alpha\Lambda/3\gamma}\right)^{\frac{n-2}{2}} \to 0$
- Non-derivative couplings $\sim m_0^2 \left(\frac{\kappa^2}{\bar{\gamma}}\right)^{\frac{n-2}{2}} = \frac{\gamma}{\alpha}\left(\frac{1}{M_p}\right)^{n-2} \left(\frac{1}{1+4\alpha\Lambda/3\gamma}\right)^{\frac{n-2}{2}} \to 0$

$$\Longrightarrow \qquad S_0[\bar{g},\phi] = \int d^4x \sqrt{-\bar{g}} \left(-\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi \right)$$

$\alpha \to \infty$ is still a massless limit but now the scalar field becomes free (as $\Lambda = 0$)!

Limit $\alpha \to \infty$: recap

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(\gamma(R-2\Lambda) + \frac{\alpha}{6}R^2 - \frac{\beta}{2}C_{\mu\nu\rho\sigma}C^{\mu\nu\rho\sigma}\right), \qquad M_p^2 = \frac{\gamma}{\kappa^2}$$

We noticed that

- A non-zero cosmological constant can affect the interaction couplings;
- When $\Lambda = 0$, the limit $\alpha \to \infty$ gives a massless self-interacting spin-0
- When $\Lambda \neq 0$ additional dependences on $1/\alpha$ appear
- Thus, when $\Lambda \neq 0$ the limit $\alpha \to \infty$ kills all the spin-0 interactions

Next, we want to consider the more interesting limit $\beta \to \infty$ in both cases $\Lambda = 0$ and $\Lambda \neq 0$

Limit $\beta \to \infty$ with $\Lambda = 0$

Action for the massive spin-2 around $f_{\mu\nu} = 0$ and $\bar{g}_{\mu\nu}$:

$$S_2[\bar{g}, f] = -S_{PF}[\bar{g}, f] - \int d^4x \sqrt{-\bar{g}} \left[\left(2f_\mu^{\ \rho} f_{\rho\nu} - f f_{\mu\nu} \right) \bar{R}^{\mu\nu} - \frac{\bar{R}}{2} \left(f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} f^2 \right) \right]$$
$$+ O(f^3),$$

When $\Lambda = 0$, the limit $\beta \to \infty$ gives

- S_{EH}-term couplings $\sim \left(\frac{\kappa^2}{\gamma} \right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p} \right)^{n-2} \to$ finite
- Mass-term couplings $\sim m_2^2 \left(\frac{\kappa^2}{\gamma} \right)^{\frac{n-2}{2}} = \frac{\gamma}{\beta} \left(\frac{1}{M_p} \right)^{n-2} \to 0$
- It is a massless limit: $m_2^2 = \frac{\gamma}{\beta} \to 0$

Typically, the massless limit in theories of Massive Gravity can lead to strong coupling even below M_p. [See reviews by Hinterbichler (2011) and de Rham (2014)]

What happens in renormalizable quantum gravity?

Brief digression on Massive Gravity with $\Lambda = 0$

Massive Gravity:

$$S_{MG} = \int d^4x \sqrt{-g}\left[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f \nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f \nabla^\rho f - \frac{m_2^2}{2}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\right] + O(f^3)$$

Naively, the limit $m_2^2 \to 0$ seems to give a massless spin-2 with 2 dofs…

The number of degrees of freedom must be preserved: Stückelberg trick

$$f_{\mu\nu} = f'_{\mu\nu} + \frac{1}{m_2}\left(\nabla_\mu A_\nu + \nabla_\nu A_\mu\right) + \frac{2}{m_2^2}\nabla_\mu\nabla_\nu\varphi,$$

Gauge symmetries:

$$\delta f'_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu,$$
$$\delta A_\mu = -m_2 \xi_\mu + \nabla_\mu \xi,$$
$$\delta\varphi = -m_2\, \xi,$$

Massless limit (2+2+1 = 5 dofs):

$$S_{MG}[f', A, \varphi] = S_{FP}^{(m_2=0)}[f'] + \int d^4x \sqrt{-g}\left(-\frac{1}{2}F^{\mu\nu}F_{\mu\nu} - 3\nabla_\rho\varphi\nabla^\rho\varphi\right) + O(f^3)$$

Possible strong coupling from helicity-0 interactions: $O(f^3) \sim \frac{1}{m_2} O(\varphi^3) \to \infty$

[See reviews by Hinterbichler (2011) and de Rham (2014)]

Limit $\beta \to \infty$ with $\Lambda = 0$: massless limit

Does a strong coupling (below M_p) arise in the strictly renormalizable quantum gravity? [first asked by Hinterbichler & Saravani (2016)]

A strong coupling in the limit $m_2^2 \to 0$ (i.e., $\beta \to \infty$) can be avoided *only* in $D = 4$!

Stückelberg decomposition for $\Lambda = 0$ and in D dimensions:

$$f_{\mu\nu} = f'_{\mu\nu} + \frac{1}{m_2}(\nabla_\mu A'_\nu + \nabla_\nu A'_\mu), \qquad A'_\mu = A_\mu + \frac{1}{m_2}\nabla_\mu \varphi$$

$$\Longrightarrow \quad S_2'[g,f] = \frac{\gamma}{2\kappa^2}\int d^D x\sqrt{-g}R + \int d^D x\sqrt{-g}\left[-\sqrt{\frac{\gamma}{\kappa^2}}G_{\mu\nu}f^{\mu\nu} + \frac{m_2^2}{2}(f_{\mu\nu}f^{\mu\nu} - f^2)\right]$$

$$= \frac{\gamma}{2\kappa^2}\int d^D x\sqrt{-g}R + \int d^D x\sqrt{-g}\left[-\sqrt{\frac{\gamma}{\kappa^2}}G_{\mu\nu}f'^{\mu\nu} + \frac{m_2^2}{2}(f'_{\mu\nu}f'^{\mu\nu} - f'^2)\right.$$

$$\left. + \frac{1}{2}F^{\mu\nu}F_{\mu\nu} + 2m_2 f'^{\mu\nu}(\nabla_\mu A'_\nu - g_{\mu\nu}\nabla^\rho A'_\rho) - 2R^{\mu\nu}A'_\mu A'_\nu\right], \qquad \frac{\gamma}{\kappa^2} = M_p^{D-2}$$

Possible strong coupling from $R^{\mu\nu}A'_\mu A'_\nu \sim \frac{1}{m_2^2} R^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi$?

Limit $\beta \to \infty$ with $\Lambda = 0$: massless limit

Make a field redefinition:

$$f'_{\mu\nu} \to f'_{\mu\nu} + a\, A'_\mu A'_\nu + b\, g_{\mu\nu}\, A'_\rho A'^\rho$$

In the massless limit $m_2^2 \to 0$ $(\beta \to \infty,\ \Lambda = 0)$ we get

$$\Longrightarrow \quad S'_2[g, f', A'] = \frac{\gamma}{2\kappa^2}\int d^D x\,\sqrt{-g}R + \int d^D x\,\sqrt{-g}\Bigg[-\sqrt{\frac{\gamma}{\kappa^2}}G_{\mu\nu}f'^{\mu\nu} + \frac{m_2^2}{2}\left(f'_{\mu\nu}f'^{\mu\nu} - f'^2\right) + \frac{1}{2}F^{\mu\nu}F_{\mu\nu}$$

$$+2m_2 f'^{\mu\nu}\left(\nabla_\mu A'_\nu - g_{\mu\nu}\,\nabla^\rho A'_\rho\right) + a\, f'^{\mu\nu}A'_\mu A'_\nu + [b(1-D) - 2a]f' A'_\rho A'^\rho$$

$$-\frac{1}{m_2^2}\left(a\sqrt{\frac{\gamma}{\kappa^2}} + 2m_2^2\right)R^{\mu\nu}A'_\mu A'_\nu - \sqrt{\frac{\gamma}{\kappa^2}}\left(\left(1-\frac{D}{2}\right)b - \frac{a}{2}\right)RA'_\rho A'^\rho$$

$$-m_2(2b(1-D) - 3a)\, A'_\mu A'_\nu \nabla^\mu A'^\nu - \frac{m_2^2}{2}\left(b^2 D(1-D) + 2ab(1-D)\right)\left(A'_\rho A'^\rho\right)^2\Bigg]$$

4 conditions to avoid strong coupling in the massless limit:

$$a\sqrt{\frac{\gamma}{\kappa^2}} + 2m_2^2 = 0, \qquad 2b(1-D) - 3a = 0,$$

$$\left(1-\frac{D}{2}\right)b - \frac{a}{2} = 0, \qquad b^2 D(1-D) + 2ab(1-D) = 0$$

can be simultaneously satisfied *only* in $D = 4$!!!

$$a = -\sqrt{\frac{\kappa^2}{\gamma}}\,2m_2^2 = -2b,$$

Limit $\beta \to \infty$ with $\Lambda = 0$: massless limit

Then, by canonically normalizing the fields, making the additional transformations

$$f'_{\mu\nu} \to f'_{\mu\nu} - \sqrt{\frac{\gamma}{4\kappa^2}}\, g_{\mu\nu}, \qquad g_{\mu\nu} \to e^{-\sqrt{2\kappa^2/3\gamma}\varphi} g_{\mu\nu},$$

we get the following action as a result of the limit $\beta \to \infty$ with $\Lambda = 0$:

$$S[g,\phi,f',A,\varphi] = S_{EH}[g] + S_0\left[\,(g - \sqrt{2k^2/\gamma} f')e^{-\sqrt{2\kappa^2/3\gamma}\varphi}, \phi\right] + S_2^{(m_2=0)}[g,f']$$

$$+ \int d^4x \sqrt{-g}\left[\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}e^{-\sqrt{2\kappa^2/3\gamma}\varphi}\nabla_\mu\varphi\nabla^\mu\varphi\right]$$

- The massive spin-2 contains 5 massless ghost-like dofs ($\pm 2,\ \pm 1,\ 0$)
- The self-interactions of $f'_{\mu\nu}$ and $h_{\mu\nu}$ do <u>not</u> vanish;
- Self-interactions of A_μ vanish; those of φ can be absorbed into a field redefinition
- Various mutual interactions survive among all fields survive

<u>Main point:</u> the limit $\beta \to \infty$ with $\Lambda = 0$ does not give rise to divergent couplings and most of the interactions survive

Limit $\beta \to \infty$ with $\Lambda \neq 0$

Action for the massive spin-2 around $f_{\mu\nu} = 0$ and $\bar{g}_{\mu\nu}$ (de Sitter):

$$S_2[\bar{g}, f] = -S_{PF}[\bar{g}, f] - \int d^4x \sqrt{-\bar{g}} \left[\left(2 f_\mu^{\rho} f_{\rho\nu} - f f_{\mu\nu} \right) \bar{R}^{\mu\nu} - \frac{\bar{R}}{2} \left(f_{\mu\nu} f^{\mu\nu} - \frac{1}{2} f^2 \right) \right]$$

$$+ \, interactions,$$

When $\Lambda \neq 0$, the limit $\beta \to \infty$ gives

- S_{EH}-term couplings $\sim \left(\frac{\kappa^2}{\gamma} \right)^{\frac{n-2}{2}} = \left(\frac{1}{M_p} \right)^{n-2} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3\gamma} \right)^{\frac{n-2}{2}} \to 0$
- Mass-term couplings $\sim m_2^2 \left(\frac{\kappa^2}{\gamma} \right)^{\frac{n-2}{2}} = \frac{\gamma}{\beta} \left(\frac{1}{M_p} \right)^{n-2} \left(\frac{1}{1+2\Lambda(2\alpha+\beta)/3\gamma} \right)^{\frac{n-2}{2}} \to 0$
- <u>NOT</u> a massless limit: $m_2^2 = \frac{\gamma}{\beta} + \frac{2}{3} \left(2 \frac{\alpha}{\beta} + 1 \right) \Lambda \to \frac{2}{3} \Lambda$

<u>**Be careful:**</u> in theories of Massive Gravity this limit for the mass is known as <u>*partially massless limit*</u> and in general may lead to strong coupling.

Brief digression on Massive Gravity with $\Lambda > 0$

Massive Gravity around de Sitter:

$$S_{MG} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f\nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f\nabla^\rho f$$
$$+\Lambda\left(f_{\mu\nu}f^{\mu\nu} - \tfrac{1}{2}f^2\right) - \tfrac{m_2^2}{2}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\Big] + \mathcal{O}(f^3)$$

Naively, the limit $m_2^2 \to \frac{2}{3}\Lambda$ seems to give a *partially massless* spin-2 with 4 dofs…

The quadratic part of the action becomes:

$$S_{FP} = \int d^4x\sqrt{-g}\Big[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f\nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f\nabla^\rho f$$
$$+\Lambda\left(f_{\mu\nu}f^{\mu\nu} - \tfrac{1}{2}f^2\right) - \tfrac{\Lambda}{3}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\Big]$$

Scalar gauge symmetry kills 1 dof: (5-1=4 dofs)

$$\delta f_{\mu\nu}(x) = \nabla_\mu\nabla_\nu\zeta(x) + \frac{\Lambda}{3}g_{\mu\nu}\zeta(x),$$

This is known as partially massless gravity where the graviton propagates 4 dofs

[Deser, Nepomechie (1984); Deser, Waldron (2001+); etc]

Brief digression on Massive Gravity with $\Lambda > 0$

Massive Gravity around de Sitter:

$$S_{MG} = \int d^4x \sqrt{-g}\Big[-\frac{1}{2}\nabla_\rho f_{\mu\nu}\nabla^\rho f^{\mu\nu} + \nabla_\rho f_{\mu\nu}\nabla^\mu f^{\rho\nu} - \nabla_\mu f \nabla_\nu f^{\mu\nu} + \frac{1}{2}\nabla_\rho f \nabla^\rho f$$

$$+\Lambda\left(f_{\mu\nu}f^{\mu\nu} - \tfrac{1}{2}f^2\right) - \tfrac{m_2^2}{2}\left(f_{\mu\nu}f^{\mu\nu} - f^2\right)\Big] + O(f^3)$$

Naively, the limit $m_2^2 \to \frac{2}{3}\Lambda$ seems to give a *partially massless* spin-2 with 4 dofs…

The number of degrees of freedom must be preserved: Stückelberg trick

$$f_{\mu\nu} = f'_{\mu\nu} + \sqrt{\frac{3}{\Lambda}}\frac{1}{\Delta}\left(\nabla_\mu\nabla_\nu\varphi + g_{\mu\nu}\frac{\Lambda}{3}\varphi\right)$$

$$\Delta \equiv m_2^2 - \tfrac{2}{3}\Lambda$$

Gauge symmetries:

$$\delta f'_{\mu\nu} = \nabla_\mu\nabla_\nu\zeta + \frac{\Lambda}{3}g_{\mu\nu}\zeta,$$

$$\delta\varphi = -\sqrt{\frac{\Lambda}{3}}\,\Delta\,\zeta,$$

Partially massless limit $\Delta \to 0$ after substitution (4+1=5 dofs):

$$S_{MG}[f',\varphi] = S_{FP}^{(\Delta=0)}[f'] + 3\int d^4x\sqrt{-g}\left(-\frac{1}{2}\nabla_\rho\varphi\nabla^\rho\varphi - \frac{m_\varphi^2}{2}\varphi^2\right) + O(f^3), \qquad m_\varphi^2 \equiv -\frac{4}{3}\Lambda$$

Possible strong coupling from φ interactions: $O(f^3) \sim \frac{1}{\Delta}O(\varphi^3) \;\to\; \infty$

[de Rham, Hinterbichler, Johnson (2018)]

Limit $\beta \to \infty$ with $\Lambda \neq 0$ $\Leftrightarrow$ partially massless limit

What happens in the strictly renormalizable theory?

(Is the strong coupling avoided as in the massless limit case with $\Lambda = 0$?)

Work in progress…

Claims:

- the renormalizability of the theory in $D = 4$ should guarantee a smooth $\beta \to \infty$ limit also when $\Lambda \neq 0$
- Then, the additional dependence on β in the couplings due to $\Lambda \neq 0$, drastically changes the $\beta \to \infty$ limit of the theory: all ghost interactions might be killed (ghost decoupling)!

…stay tuned…

Possible implications?

- If α, β are positive, then in the high-energy limit we expect

$$\alpha \to 0, \qquad \beta \to \infty$$

- Question: can our discussion on $\beta \to \infty$ be useful to understand the high-energy behavior of the spin-2 ghost?

- If YES, then the presence of a cosmological constant Λ might affect the high-energy behavior of the theory non-trivially (ghost decoupling at high energies ?)

- Most of the current approaches to the ghost problem are based on a QFT formulation in Minkowski background and 'extrapolations' to curved backgrounds.

  Does a non-zero Λ affect non-trivially some of those approaches?

Hopefully some material for discussions…

Thank you
for
your attention!