Quantum Integrable Black Holes

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Abstract and outline

It is common to assume that quantum gravity belongs at the Planck scale, but a possibly much larger width for the ground state emerges in the (non-perturbative) quantisation of the Oppenheimer-Snyder model of dust collapse that naturally recovers Bekenstein's area law. The effective geometry for such quantum black holes can then be obtained from coherent states which describe integrable singularities without inner horizons. The extension to quantum (differentially) rotating black holes with similar properties is also described.

Complementary quantum gravity views at self-gravitating objects:

- Planck scale in (perturbative EFT for) gravity and gravitational collapse

1. Macroscopic quantum width in (non-perturbative) gravitational collapse
2. Quantum (non-perturbative) coherent state for black hole geometry without Cauchy horizons

↓

3. Quantum (*integrable**) black holes

* V.N. Lukash, V.N. Strokov, IJMPA 28 (2013) 1350007 [arXiv:1301.5544]

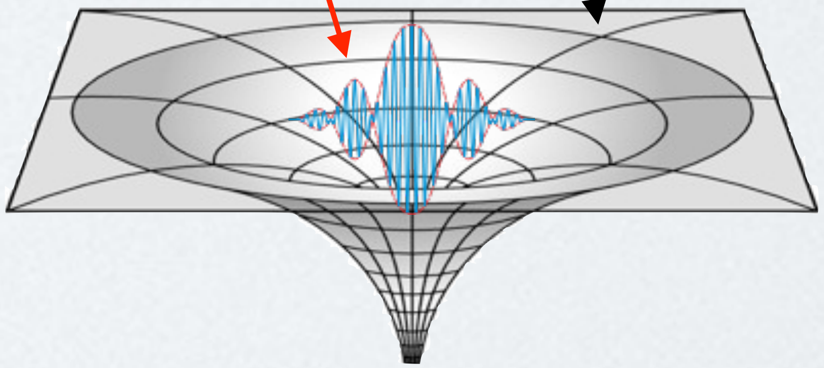
Preamble - QG and Planck scale

Planck scale:

$$m = m_{\mathrm{p}} \equiv \sqrt{\frac{c\,\hbar}{G_{\mathrm{N}}}}$$

$$\lambda_{\mathrm{C}} = \ell_{\mathrm{p}} \equiv \sqrt{\frac{\hbar\, G_{\mathrm{N}}}{c^3}}$$

$$\frac{\hbar}{m\,c} \equiv \lambda_{\mathrm{C}} \sim R_{\mathrm{H}} \equiv \frac{2\,G_{\mathrm{N}}\,m}{c^2}$$



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$$\frac{\hbar}{m\,c} \equiv \lambda_{\mathrm{C}} \sim R_{\mathrm{H}} \equiv \frac{2\,G_{\mathrm{N}}\,m}{c^2}$$

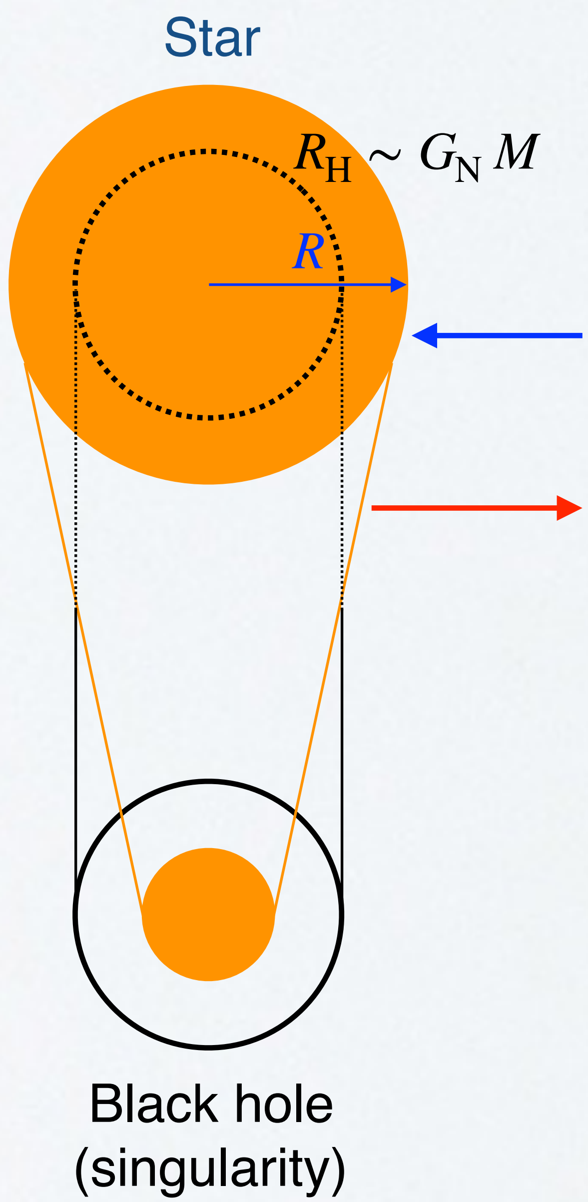
Is Quantum Gravity confined at the Planck scale?

In (1-loop corrected) EFT:

$$\frac{S_{\mathrm{eff}}}{\hbar} = \frac{1}{\hbar}\left(-\int \frac{d^4x\,\sqrt{-g}\,R}{16\,\pi\,G_{\mathrm{N}}} + S_{\mathrm{m}} + \hbar\,\Gamma^{(1)}\right) \sim -\frac{L^2}{\ell_{\mathrm{p}}^2} + \frac{M\,L}{m_{\mathrm{p}}\,\ell_{\mathrm{p}}} + \Gamma^{(1)} \sim \frac{L^2}{\ell_{\mathrm{p}}^2}\left(-1 + \frac{M\,\ell_{\mathrm{p}}}{m_{\mathrm{p}}\,L} + \frac{\ell_{\mathrm{p}}^2}{L^2}\,\Gamma^{(1)}\right)$$

$$\frac{G_{\mathrm{N}}\,M}{L} \sim 1$$

- BH form from collapse - the semiclassical (classical background) view:

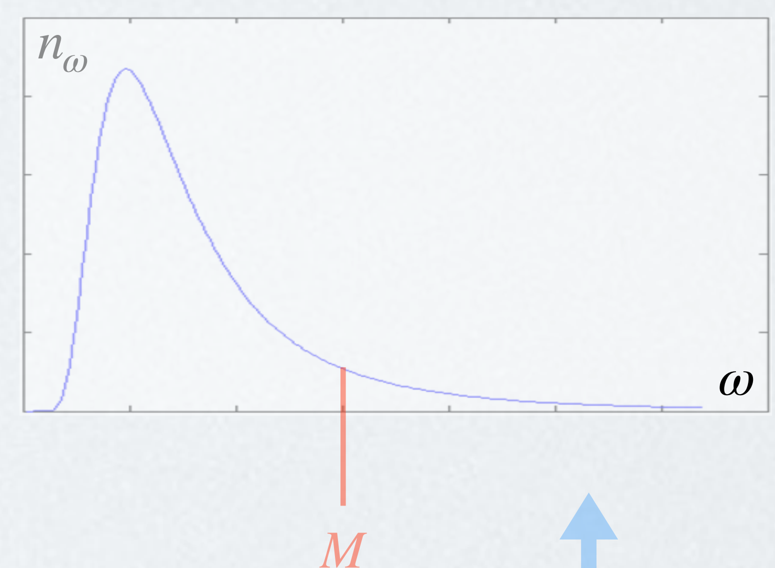


- Classical star + $|\,vacuum\ \text{fluctuations}\rangle$
- Classical inner and outer geometry + $|\,vacuum\ \text{fluctuations}\rangle$

Time-dependent
(gravitational blue/redshift) → $|\,\text{Thermal excitations}\rangle$

BH temperature: $T_\mathrm{H} \sim m_\mathrm{p}\,\dfrac{m_\mathrm{p}}{M}$

BH decay rate: $\dfrac{dM}{dt} \sim -\dfrac{m_\mathrm{p}^2}{M^2}$



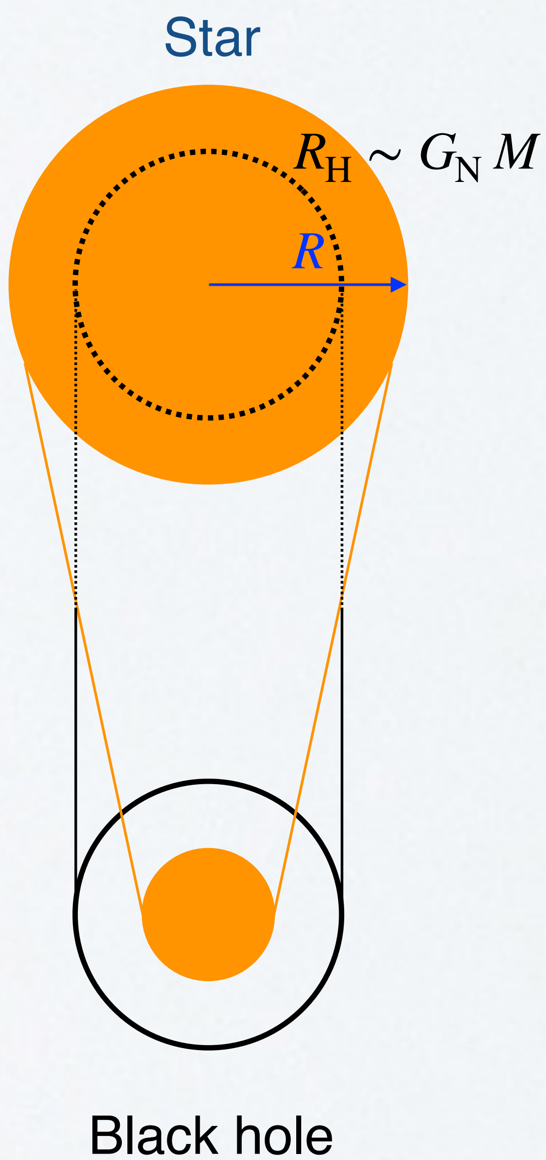
Microcanonical corrections*: $\dfrac{dM}{dt} \sim -\left(\dfrac{M}{m_\mathrm{p}}\right)^{\alpha}$

"quantum hair"

* R.C., B. Harms, PRD 58 (1998) 044014 [gr-qc/9712017]

Preamble - Gravitational collapse and black holes

- BH form from collapse - the quantum view:



- $|\text{matter}\rangle$ ~ very large number of SM particles ($M_\odot \sim 10^{57}$ neutrons)
- $|\text{gravity}\rangle$ ~ very large number of gravitons* ($N_G \sim M_\odot^2 \sim 10^{76}$)

- $|\text{gravity}\rangle$ always entangled with $|\text{matter}\rangle$ $\Longleftrightarrow$ "quantum hair" **

Dynamics

$$|\mathbf{g}\ \phi\rangle = \sum_{ij} C_{ij} |\mathbf{g}_i\rangle |\phi_j\rangle \quad \not\Rightarrow \quad \left(\sum_{ab} c_{ab} |\mathbf{g}_a\rangle |\phi_b\rangle\right) \left(\sum_{AB} c_{AB} |\mathbf{g}_A\rangle |\phi_B\rangle\right)$$

$$\hat{H}^\mu |\mathbf{g}\ \phi\rangle = 0$$

BH interior — BH exterior

* J.D. Bekenstein, PRD 7 (1973) 2333

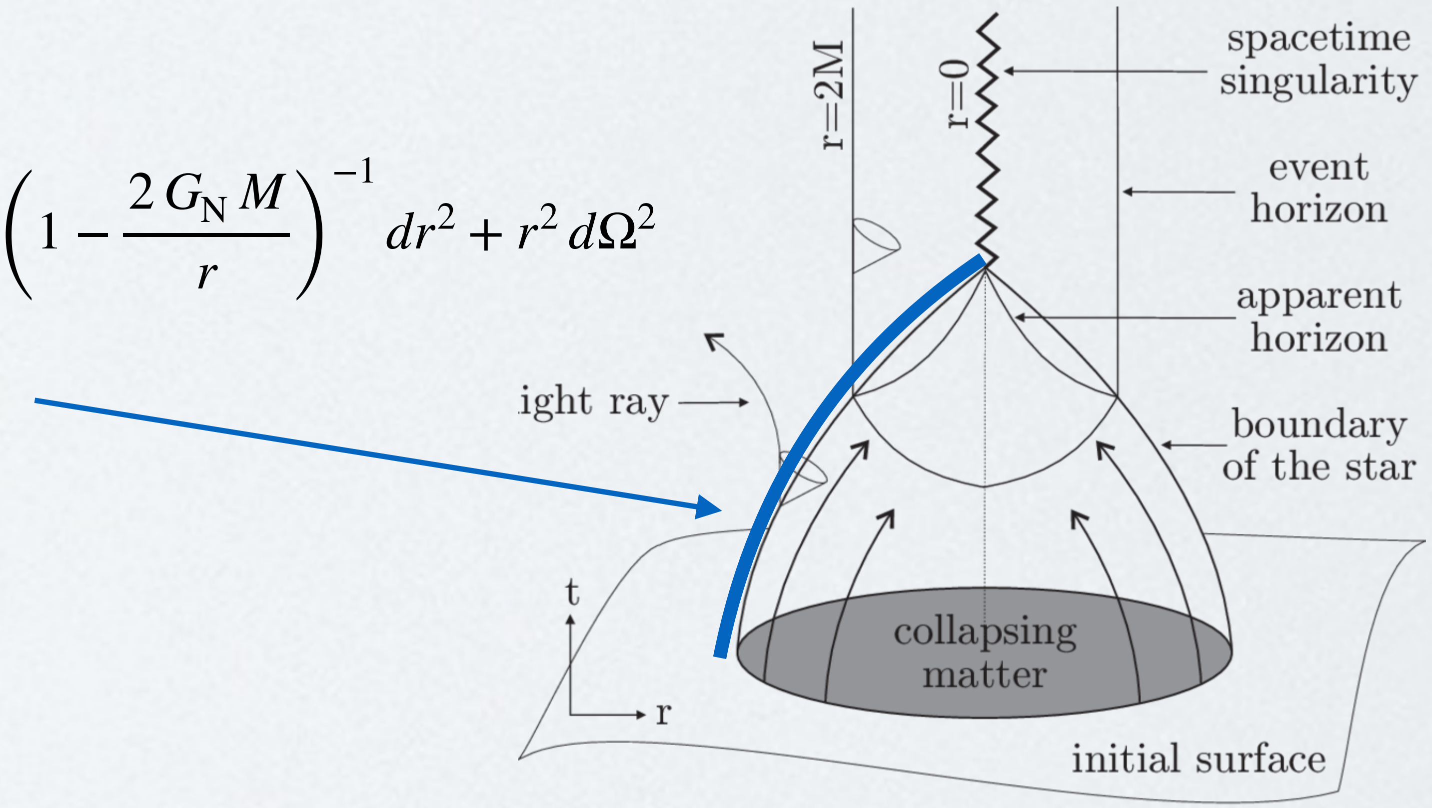
** X. Calmet, R.C., S.D.H. Hsu, F. Kuipers, PRL 128 (2022) 111301 [arXiv:2110.09386]

1 - Quantum dust core

- Role of non-linearity: collapsing ball of dust [1]

$$ds^2 = -\left(1 - \frac{2\,G_N\,M}{r}\right) dt^2 + \left(1 - \frac{2\,G_N\,M}{r}\right)^{-1} dr^2 + r^2\,d\Omega^2$$

$$\left(\frac{dR}{d\tau}\right)^2 + 1 - \frac{2\,G_N\,M}{R} \simeq \frac{E^2}{\mu^2}$$



[1] R.C., *Quantum dust cores of black holes* [arXiv:2304.06816]

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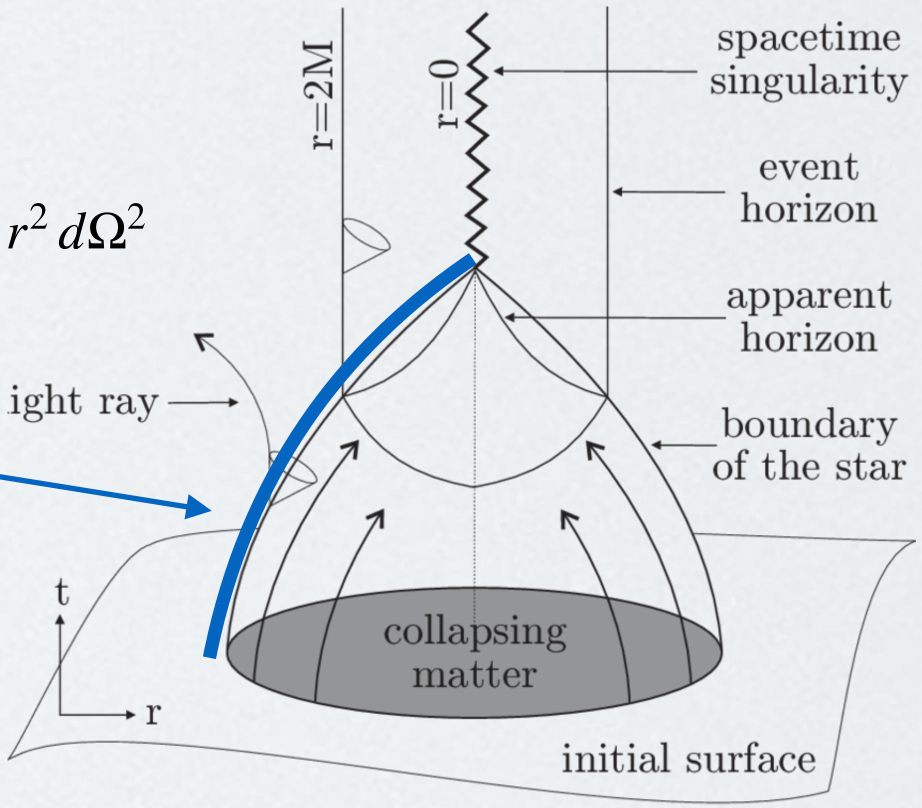

- Effective one-body ($\mu \sim M$) Hamiltonian [2,3]:

$$H \equiv \frac{P^2}{2\,\epsilon\,M} - \frac{G_N\,\epsilon\,M^2}{R} = \frac{\epsilon\,M}{2}\left(\frac{E^2}{\epsilon^2\,M^2} - 1\right) \equiv \mathscr{E}$$

$$\downarrow$$

$$H \simeq \frac{P^2}{2\,M} - \frac{G_N\,M^2}{R} = \frac{M}{2}\left(\frac{E^2}{M^2} - 1\right) \simeq \mathscr{E}$$

[2] R.C., *A quantum bound for the compactness,* EPJC 82 (2022) 1 [arXiv:2103.14582]

[3] R.C., R. Da Rocha, P. Meert, L. Tabarroni, W. Barreto, *Configurational entropy of black hole quantum cores*, CQG 40 (2023) 075014 [arXiv:2206.10398]

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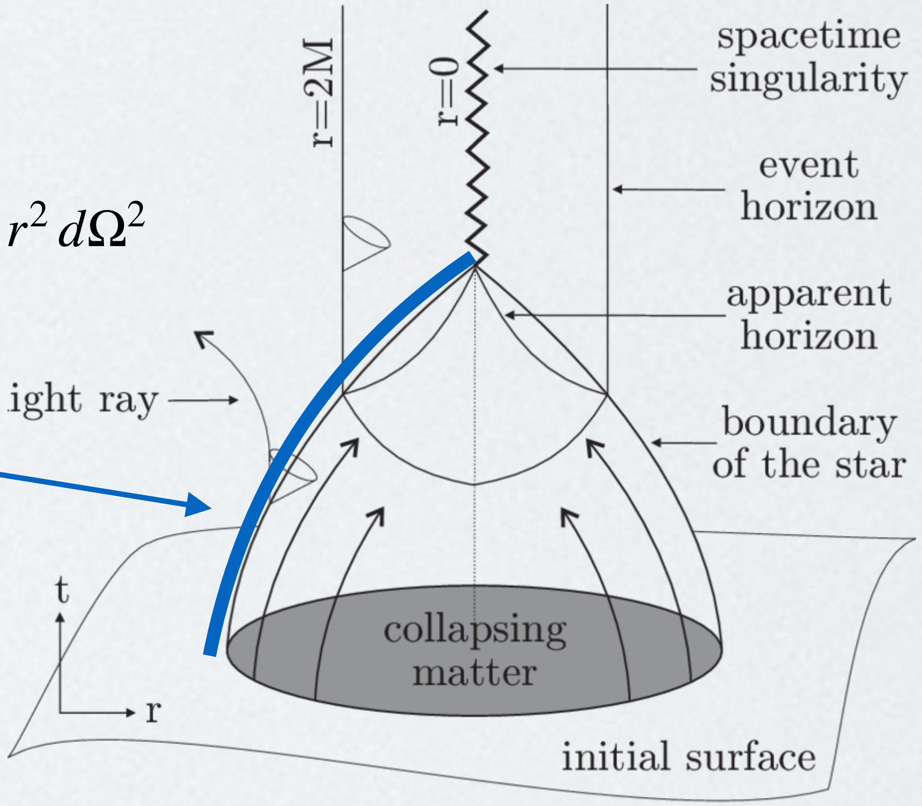


- Dust particle Hamiltonian:

$$H \equiv \frac{P^2}{2\,\mu} - \frac{G_N\,\mu\,M}{R} = \frac{\mu}{2}\left(\frac{E^2}{\mu^2} - 1\right) \equiv \mathscr{E}$$

- Schrödinger equation: $\hat{H}\,\Psi_n = \mathscr{E}_n\,\Psi_n$

- Spectrum of bound states ($n \geq 1$):

“GR” [2]

$$\frac{\mathscr{E}_n}{\mu} \simeq -\frac{G_N^2\,\mu^3\,M}{2\,\hbar^2\,n^2} = -\frac{1}{2\,n^2}\left(\frac{\mu\,M}{m_p^2}\right)^2 = \frac{1}{2}\left(\frac{E_n^2}{\mu^2} - 1\right)$$

$$R_n \equiv \langle \Psi_n | R | \Psi_n \rangle \simeq n^2\,\ell_p \left(\frac{m_p^3}{\mu^2\,M}\right)$$

Newtonian spectrum

[2] R.C., *A quantum bound for the compactness*, EPJC 82 (2022) 1 [arXiv:2103.14582]

1 - Quantum dust core

- Allowed spectrum * [1,2]: $0 \leq \frac{E_n^2}{M^2} \simeq 1 - \frac{1}{n^2}\left(\frac{\mu M}{m_p^2}\right)^2$ → $n \geq N_M \sim \frac{\mu M}{m_p^2}$

$$N_G = \frac{M}{\mu} N_M \sim \frac{M^2}{m_p^2}$$

$$R_n \geq R_{N_M} \sim G_N M = \ell_p \frac{M}{m_p}$$

- "Energy" levels: $|\mathscr{E}_{n+1} - \mathscr{E}_n| \simeq m_p \frac{m_p}{M} \ll m_p$

$$|E_{n+1} - E_n| \simeq m_p$$

- Bounded compactness: $\frac{G_N M}{R_n} \lesssim 1$

↕

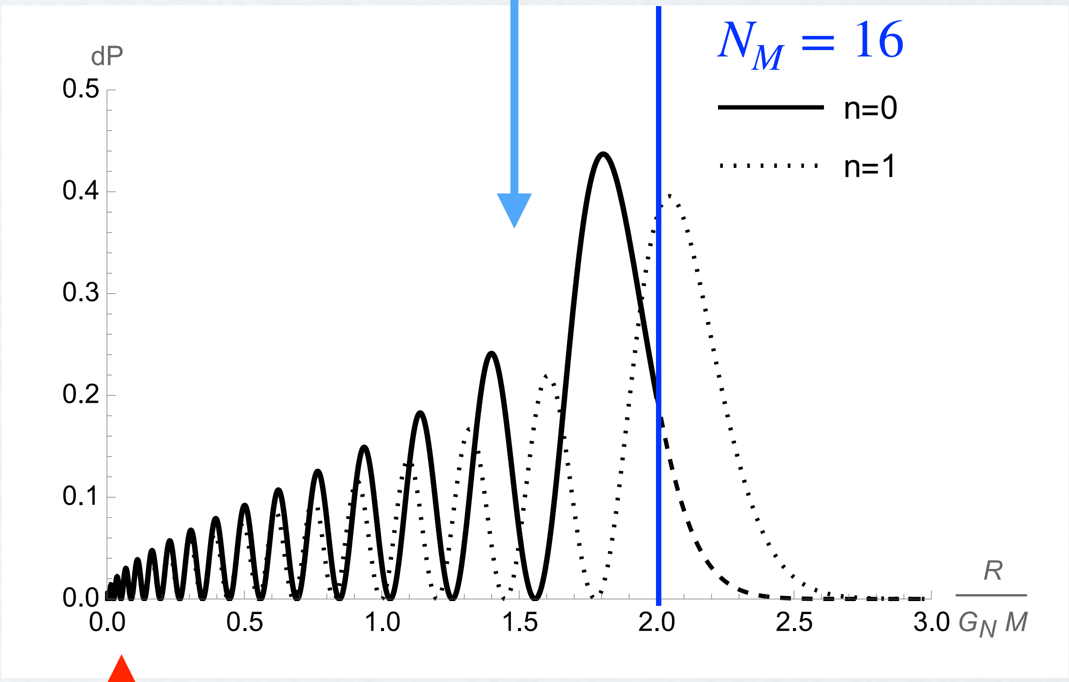
Semiclassical "bounce" (~ BH-to-WH transition)

↕

non-linearity

↕

* *Classicalization* ~ GUP in action ~ Bekenstein's (area) law



$$R_{n=1} \sim \ell_p \left(\frac{m_p}{M}\right)^3$$

[1] R.C., *Quantum dust cores of black holes* [arXiv:2304.06816]
[2] R.C., *A quantum bound for the compactness,* EPJC 82 (2022) 1 [arXiv:2103.14582]

2 - Coherent state for classical geometry

- SdS geometry [4]:

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2$$

$$f = 1 + 2\,V_{\rm SdS} = 1 - \frac{2\,G_{\rm N}\,M}{r} - \frac{\Lambda\,r^2}{3}$$

$$V_{\rm SdS} = V_M + V_\Lambda = -\frac{G_{\rm N}\,M}{r} - \frac{\Lambda\,r^2}{6}$$

- Horizons ($f = 0 \leftrightarrow 2\,V = -1$):

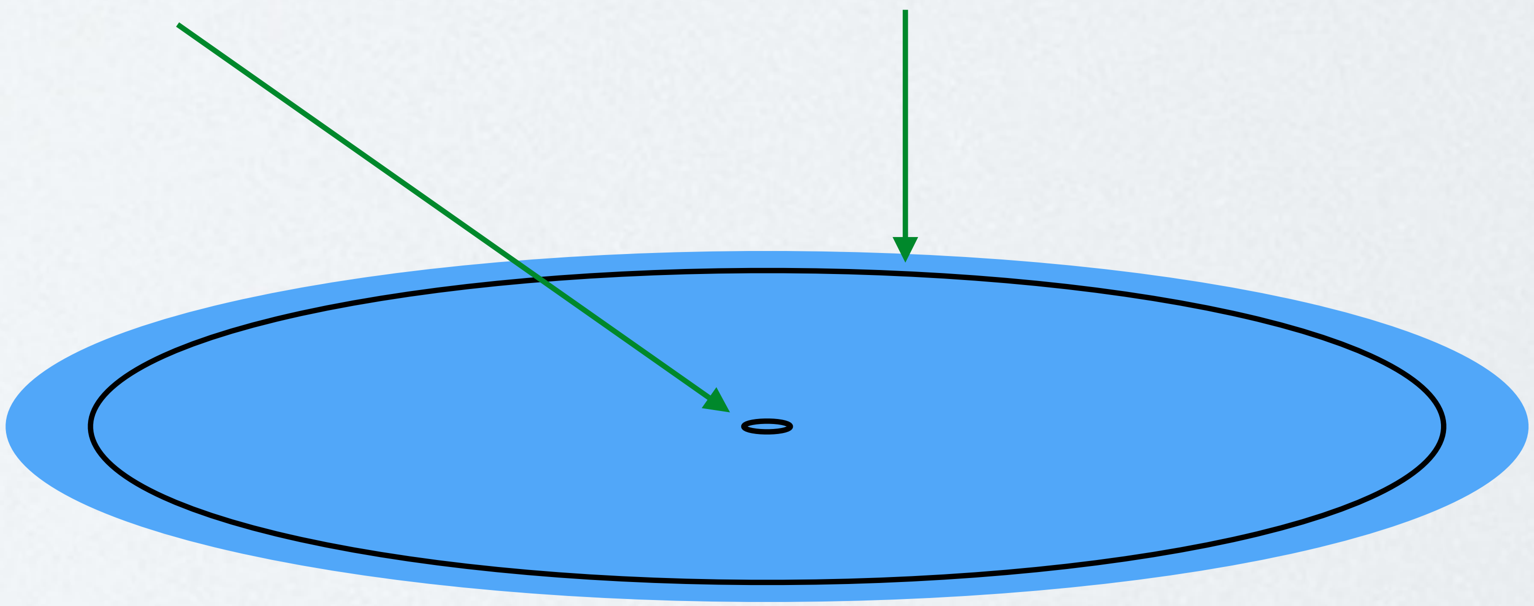
$1/\sqrt{\Lambda} \gg G_{\rm N}\,M$

Cosmological horizon

$$H^{-1} = L \simeq \sqrt{\frac{3}{\Lambda}}$$

Localised source gravitational radius

$$R_{\rm H} \simeq 2\,G_{\rm N}\,M$$



[4] A. Giusti, S. Buffa, L. Heisenberg, R.C., *A quantum state for the late* Universe, PLB 826 (2022) 136900 [arXiv:2108.05111]

2 - Coherent state for classical geometry

- "Effective" massless scalar field in Minkowski (~ true QG vacuum $\hat{a}_k\,|\,0\rangle = 0$ *):

$$\left[-\frac{\partial^2}{\partial t^2}+\frac{1}{r^2}\,\frac{\partial}{\partial r}\left(r^2\,\frac{\partial}{\partial r}\right)\right]\Phi(t,r)=0$$

$$u_k(t,r) = e^{-i\,k\,t}\,j_0(k\,r)$$

$$4\,\pi\int_0^\infty r^2\,dr\,j_0(k\,r)\,j_0(p\,r) = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Normal mode expansion of operators:

$$\hat{\Phi}(t,r) = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{\hbar}{2\,k}}\left[\hat{a}_k\,u_k(t,r)+\hat{a}_k^\dagger\,u_k^*(t,r)\right]$$

$$\left[\hat{\Phi}(t,r),\hat{\Pi}(t,s)\right] = \frac{i\,\hbar}{4\,\pi\,r^2}\,\delta(r-s)$$

$$\hat{\Pi}(t,r) = i\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{\hbar\,k}{2}}\left[\hat{a}_k\,u_k(t,r)-\hat{a}_k^\dagger\,u_k^*(t,r)\right]$$

$$\left[\hat{a}_k,\hat{a}_p^\dagger\right] = \frac{2\,\pi^2}{k^2}\,\delta(k-p)$$

- Coherent state: $\hat{a}_k\,|\,g\rangle = g(k)\,e^{i\,\gamma_k(t)}\,|\,g\rangle$

$$\langle g\,|\,\hat{\Phi}(t,r)\,|\,g\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\sqrt{\frac{2\,\ell_{\rm p}\,m_{\rm p}}{k}}\,g(k)\,\cos[\gamma_k(t)-k\,t]\,j_0(k\,r)$$

* Metric ~ causal structure ~ gravity emerges from "excitations" along with matter (~ LQG, Regge calculus, etc…)

2 - Coherent state for classical geometry

- "Classical" coherent state: $$\sqrt{\frac{\ell_{\rm p}}{m_{\rm p}}}\langle g\,|\,\hat{\Phi}(t,r)\,|\,g\rangle = V(r) = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,\tilde{V}(k)\,j_0(k\,r)$$

Single mode occupation number: $$g(k) = \sqrt{\frac{k}{2}}\,\frac{\tilde{V}(k)}{\ell_{\rm p}}$$

$$\gamma_k = k\,t$$ ← virtual non-propagating modes *

- Total occupation number (must be finite!) ~ distance from vacuum:

$$|\,g\rangle = e^{-N_{\rm G}/2}\,\exp\left\{\int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,g(k)\,\hat{a}_k^\dagger\right\}|\,0\rangle$$

$$N_{\rm G} = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,g^2(k) < \infty$$

$$\langle k\rangle = \int_0^\infty \frac{k^2\,dk}{2\,\pi^2}\,k\,g^2(k) < \infty$$

* Potentials in QFT are generated by virtual / non propagating modes (same for spherical symmetry)

2 - Coherent state for classical geometry

- Localised source: $V_M = -\frac{G_N M}{r}$ $\qquad \tilde{V}_M = -4\pi G_N \frac{M}{k^2}$

- Mass scaling [5]:

Divergences

$$N_M = 4\frac{M^2}{m_p^2}\int_0^{\infty}\frac{dk}{k} \longrightarrow 4\frac{M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}}\frac{dk}{k} = 4\frac{M^2}{m_p^2}\ln\left(\frac{k_{UV}}{k_{IR}}\right)$$

- Compton length scaling:

$$\langle k\rangle = 4\frac{M^2}{m_p^2}\int_0^{\infty}dk \longrightarrow 4\frac{M^2}{m_p^2}\int_{k_{IR}}^{k_{UV}}dk = 4\frac{M^2}{m_p^2}\left(k_{UV}-k_{IR}\right)$$

- Quantum core (BH = ground state): $k_{UV}^{-1} \simeq R_{N_M} \simeq G_N M$

* Cut-offs = existence condition for quantum state: $g(k < k_{IR}) = g(k > k_{UV}) \simeq 0$!

[5] R.C., *Geometry and thermodynamics of coherent quantum black holes,* IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

2 - Coherent state for classical geometry

- Localised source: $V_M = -\frac{G_N M}{r}$

- Localised source in dS: $k_{UV}^{-1} \simeq R_{N_M} \simeq G_N M$

$k_{IR}^{-1} \simeq L$

$$V_{QM} \equiv \sqrt{\frac{\ell_p}{m_p}} \langle g | \hat{\Phi}(t, r) | g \rangle \simeq V_M(r)$$

(*Observable?*) quantum hair

$$V_{QM} = V_M(r) \left\{ 1 - \left[1 - \frac{2}{\pi} \mathrm{Si}\left(\frac{r}{R_{N_M}} \right) \right] \right\}$$

$$\mathrm{Si}(x) = \int_0^x j_0(z)\, dz$$

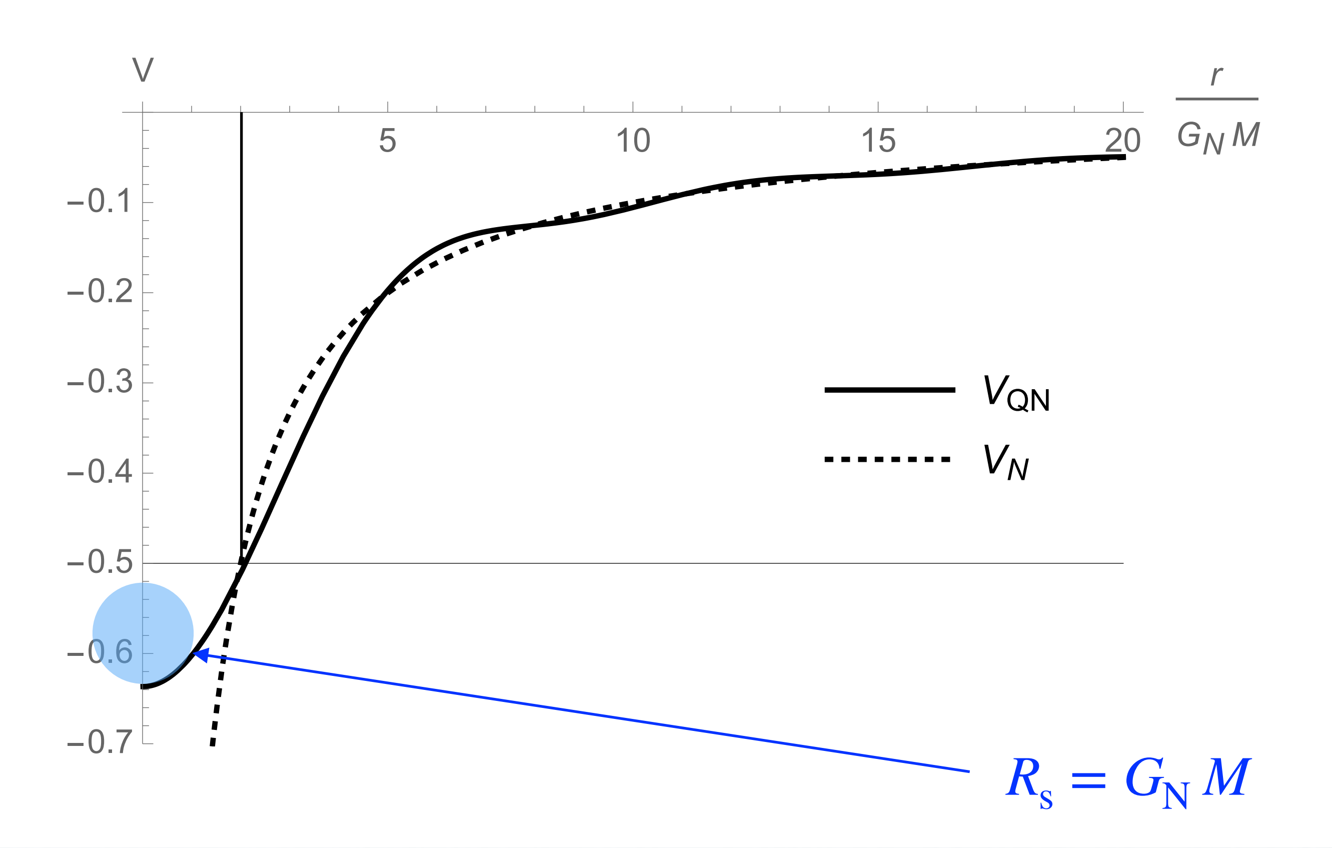
3 - Quantum black holes

- Corpuscular scaling laws:

$$N_M \sim \frac{M^2}{m_{\rm p}^2} \ln\left(\frac{R_\infty}{R_M}\right)$$

$$\lambda_M \simeq \frac{N_M}{\langle k \rangle} \sim R_{\rm H} \sim G_{\rm N}\, M$$

cutoffs → proper $g(k)$ ~ time-dependent "quantum hair"

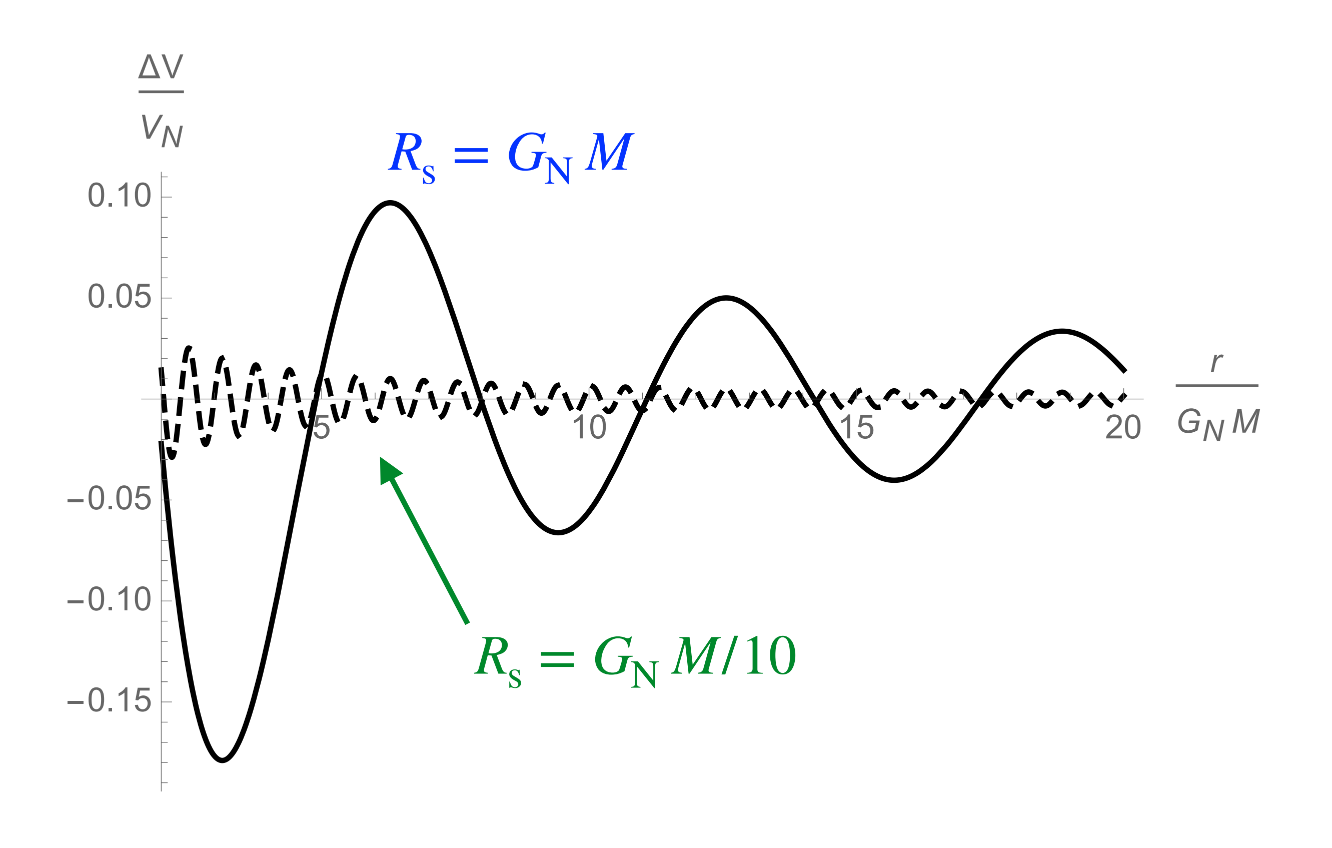


- Quantum metric [5]:

$$ds^2 \simeq -\left(1 + 2\,V_{\rm QM}\right) dt^2 + \frac{dr^2}{1 + 2\,V_{\rm QM}} + r^2\, d\Omega^2$$

Integrable singularity without inner horizon

$$R^2 \sim R_{\mu\nu\alpha\beta}\, R^{\mu\nu\alpha\beta} \sim r^{-4}$$



[5] R.C., IJMPD 31 (2022) 2250128 [arXiv:2103.00183]

3 - Quantum black holes

- Spherical *integrable singularity* without inner horizon [5,6]

$$\rho_{\text{eff}} \sim |\Psi^2(r)| \sim r^{-2} \qquad m(r) \sim \int_0^r \rho(x)\, x^2\, dx < \infty$$

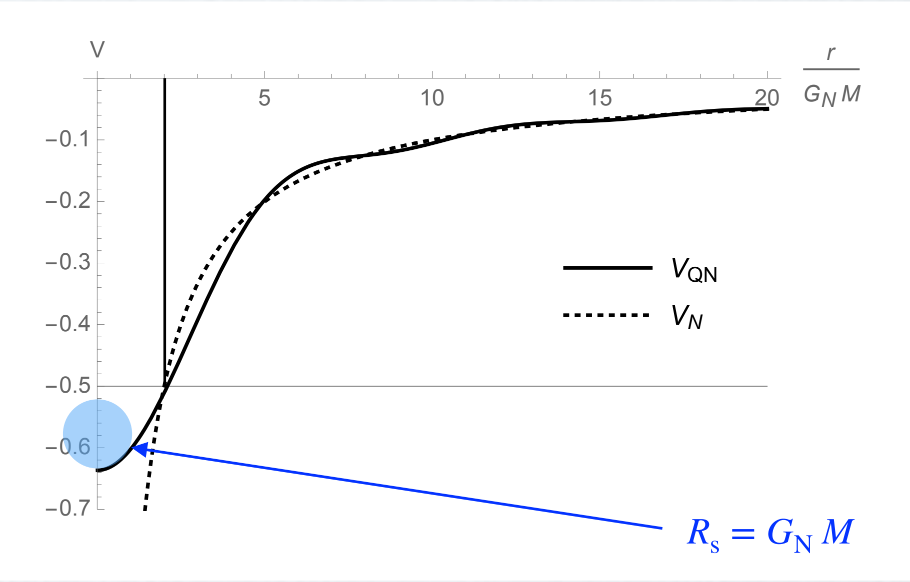
$$p^r_{\text{eff}} \sim -\rho_{\text{eff}} \sim -r^{-2}$$

$$p^t_{\text{eff}} \sim r^0$$

$m(r) \sim r$

$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$

$\Delta(r \sim 0) < 0$



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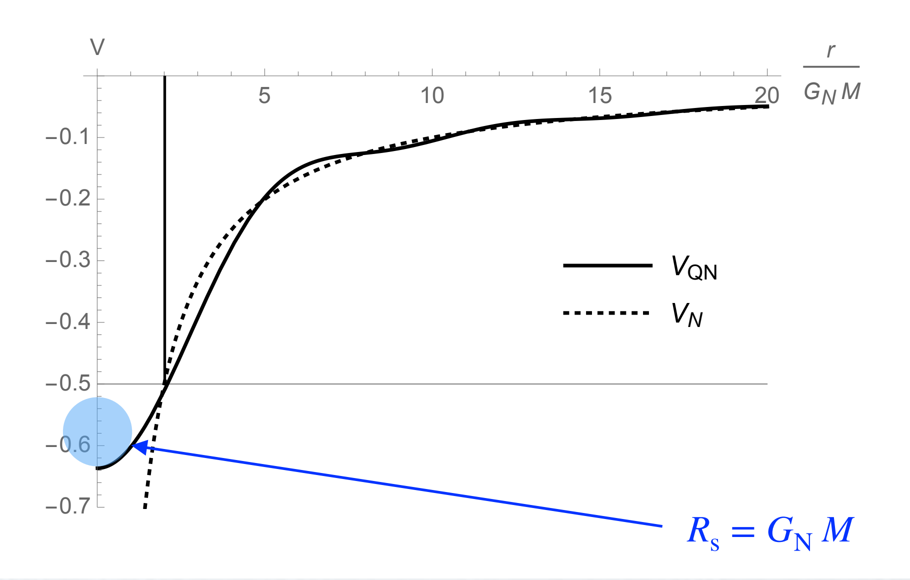


$m(r) \sim r$

$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$

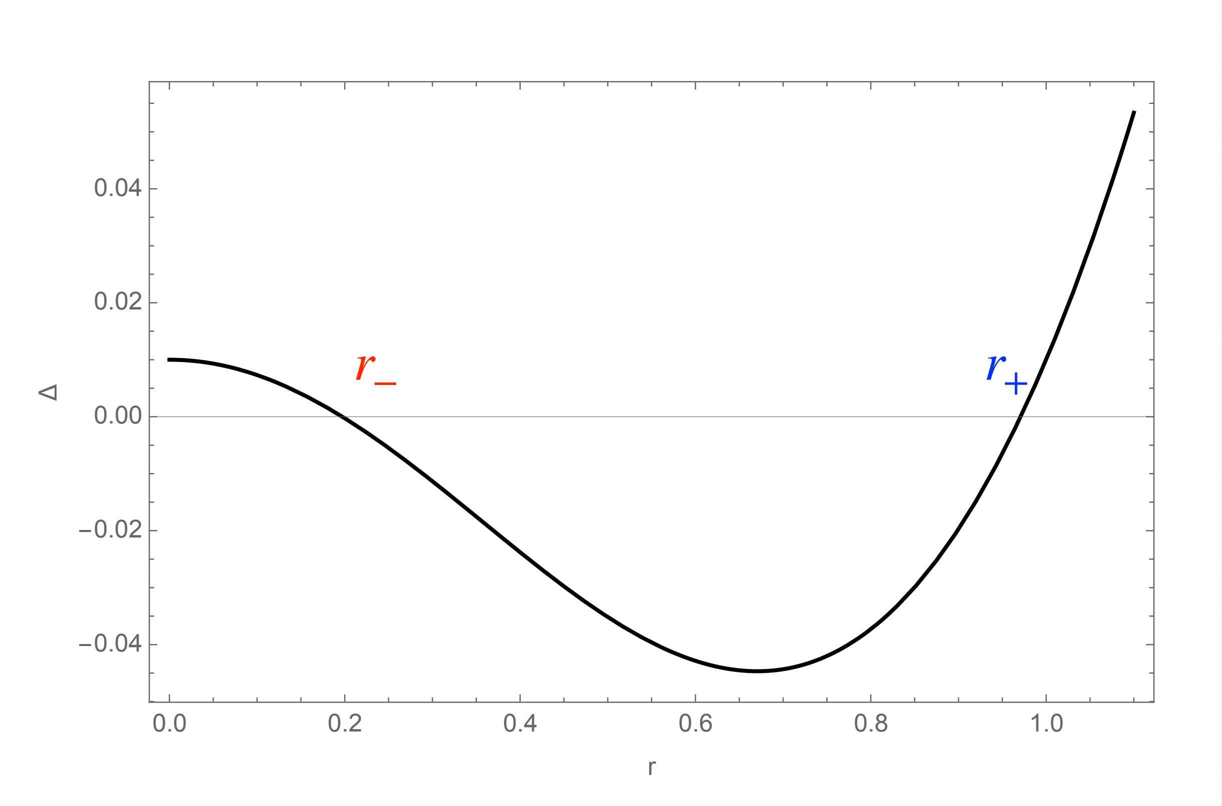
$\Delta(r \sim 0) < 0$

Regular (classical) black holes:

$$\rho \sim r^0 \quad \Longrightarrow \quad m \sim r^3$$

$$\Delta = r_\pm^2 - 2\, r_\pm\, m(r_\pm) = 0$$

$\Delta(r \sim 0) \sim r^2 > 0$



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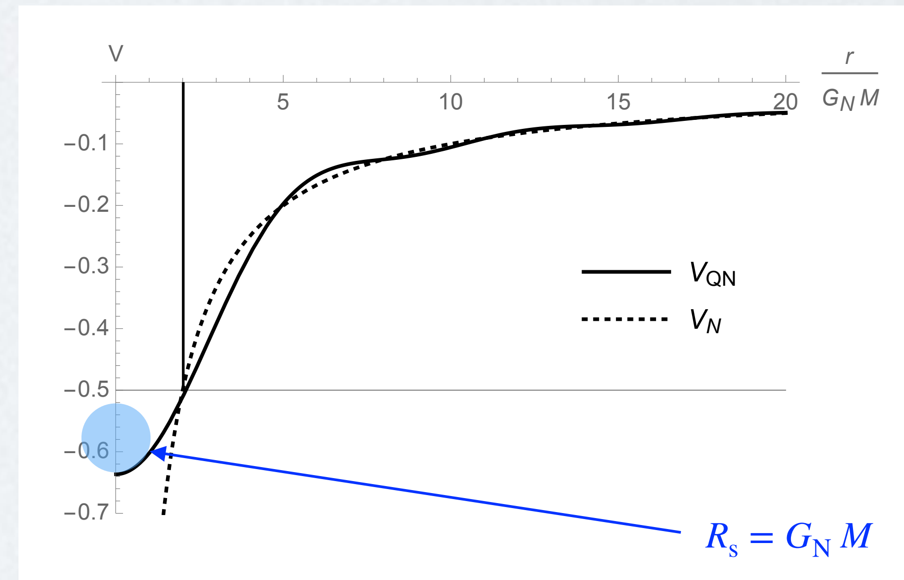
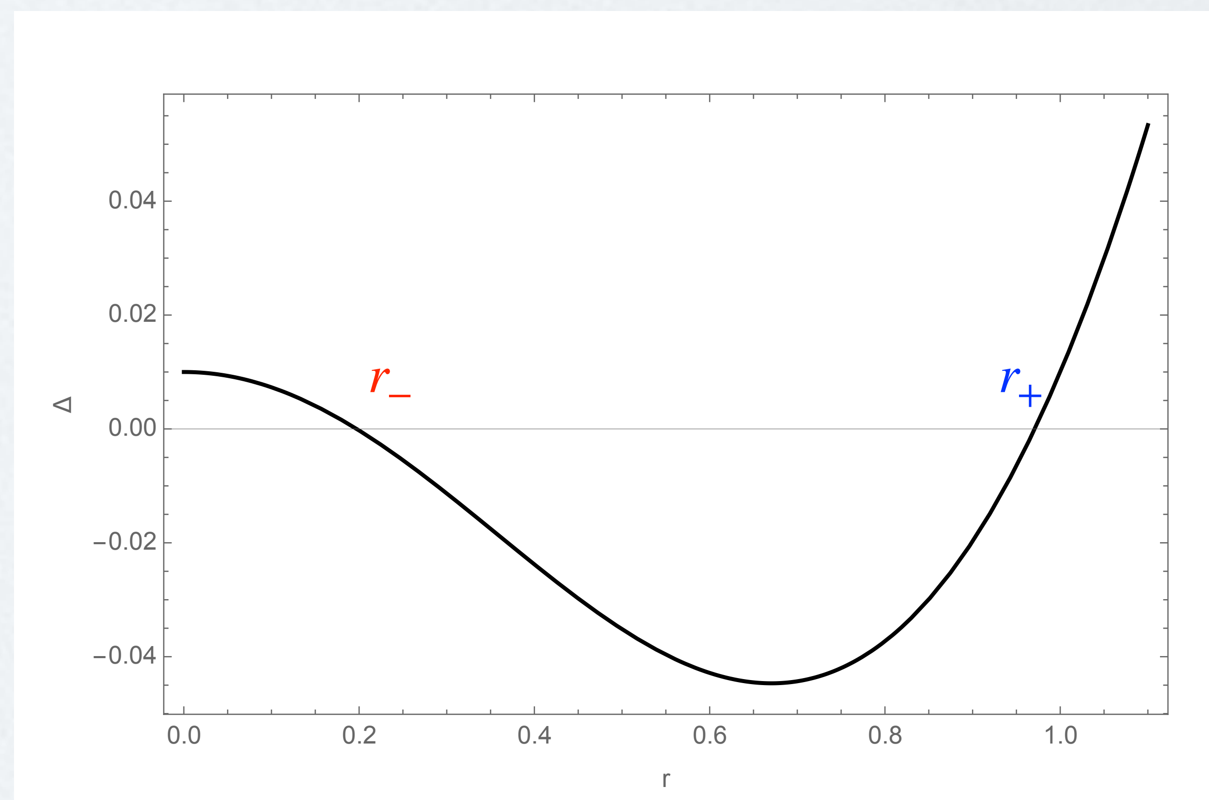


- Rotating *integrable singularity* without inner horizon [7]

$$m(r) \sim r$$

$$\Delta = a^2 - 2\, r_\pm\, m(r_\pm) + r_\pm^2 = 0$$

$$\Delta(0) = a^2 > 0$$



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[7] R.C., A. Giusti, J. Ovalle, *Quantum rotating black holes* [arXiv:2303.02713]

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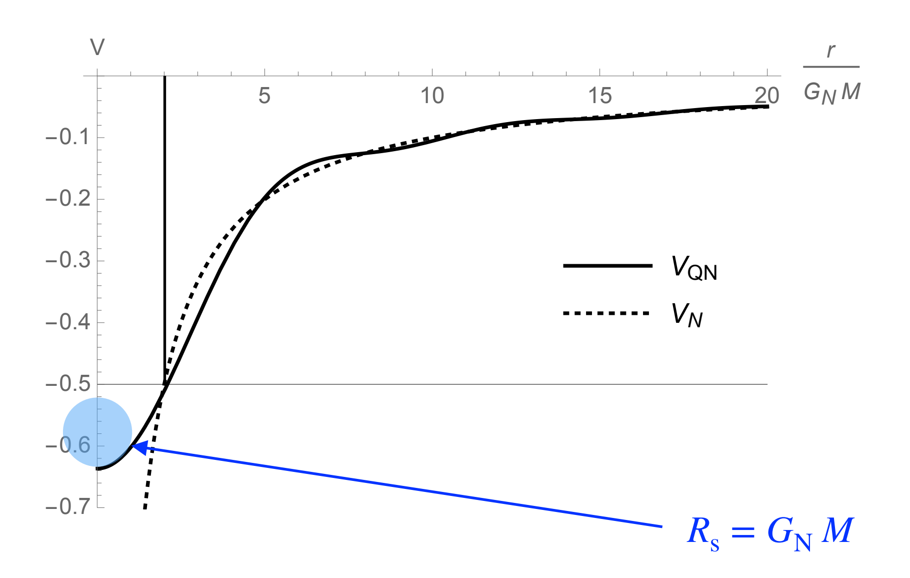


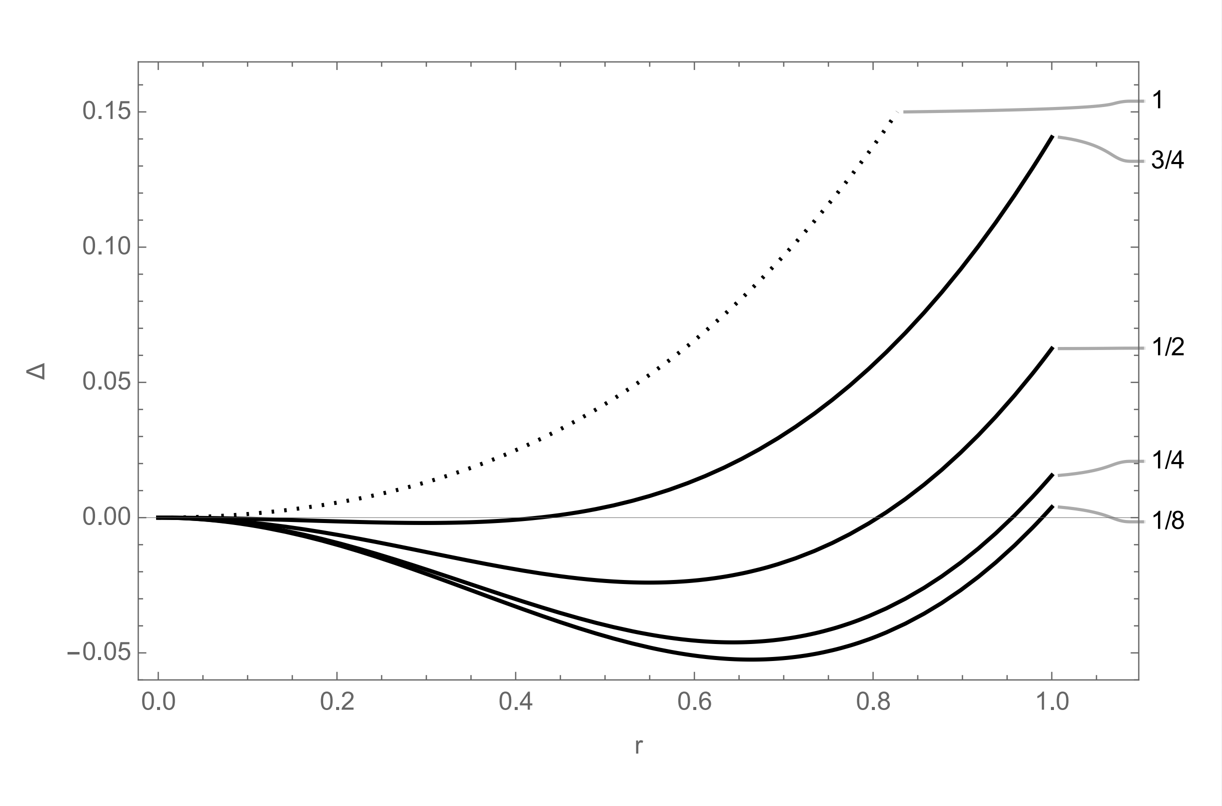
- Rotating *integrable singularity* without inner horizon [7]

$$m(r) \sim r$$

$$a(r) \sim r^\alpha \qquad \alpha \geq 1$$

$$\Delta = a^2(r_H) - 2\, r_H\, m(r_H) + r_H^2 = 0$$

$$\Delta(0) = 0$$



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Conclusions

- Black holes as (macroscopic) quantum objects (*ground state* far from vacuum)
- Singularity is not resolved (integrable “fuzzy” geometry)
- Exterior quantum hair (from core size)
- No Cauchy horizon (also for electrically charged black holes)
- No Cauchy horizon for rotating black holes
- Effective cosmological DM