LMU LUDWIG-MAXIMILIANS-UNIVERSITÄT MÜNCHEN

ARNOLD SOMMERFELD CENTER FOR THEORETICAL PHYSICS

CONFORMAL SYMMETRY IN THE STANDARD MODEL AND ITS SYMBIOSIS WITH GRAVITY

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Based on

- G.K., Mikhail Shaposhnikov, Andrey Shkerin, Sebastian Zell: 2106.13811 [hep-th]
- G.K., Mikhail Shaposhnikov, Andrey Shkerin, Sebastian Zell: 2106.13811 [hep-th]
- G.K., Mikhail Shaposhnikov, Sebastian Zell: 2203.09534 [hep-ph]
- G.K., Mikhail Shaposhnikov, Sebastian Zell: to appear

as well as

- G.K., Alexander Monin: 1510.08042 [hep-th]
- G.K., Javier Rubio: 1606.08848 [hep-ph]
- G.K., Vladimir Kazakov, Mikhail Shaposhnikov: 1908.04302 [hep-th]
- G.K., Marco Michel, Javier Rubio: 2006.11290 [hep-th]

Outline

- Introduction and (phenomenological) motivation
- Relevance of scale/conformal symmetry
- The role of gravity
- Concluding remarks

Introduction and (phenomenological) motivation

The Standard Model (SM) of particle physics (circa '70s) is THE success story

- Description of a plethora of phenomena in the microcosm
- Its last missing piece, the Higgs boson was observed in July 2012, $\sim$ 11 years ago!
- So far no convincing deviations from the SM have been observed at particle physics experiments
- Moreover, the SM could be a self-consistent effective field theory up to very high energies ($\sim M_P$)

Introduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Introduction and (phenomenological) motivation

Do we have in our hands the final theory of Nature!?

Compelling indications that the answer is negative!

Introduction and (phenomenological) motivation

Experimental point of view

The SM (plus gravity) fails to accommodate in its context well established observational facts

- Neutrino physics
- Dark matter
- Baryon asymmetry of the Universe
- Homogeneity and isotropy of the Universe at large scales

Introduction and (phenomenological) motivation
Theoretical point of view

The SM suffers from

- Landau Pole(s) associated with the $U(1)$ & Higgs sectors, but **@** energies $\gg M_P$, so usually swept under the "quantum gravity carpet"
- Strong-CP problem
- Cosmological Constant issue
- Hierarchy issue (incredible smallness of Higgs mass M_H as compared to M_P)

Not a threat to its self-consistency
$\Rightarrow$ some pieces of the puzzle are not understood.

Various attempts to go beyond the SM

- (low-energy) Supersymmetry [Fayet '75, '77 & Witten '81 & Dimopoulos, Georgi '81 & Ibanez, Ross '81]
- Compositeness [Weinberg '76, '79 & Susskind '79]
- Large extra dimensions [Arkani-Hamed, Dimopoulos, Dvali '98 & Randall, Sundrum '99]

Distinct experimental signatures right above the electroweak scale differentiate them from the SM

So far no convincing deviations from the SM have been observed at particle physics experiments, yet

Where to look?

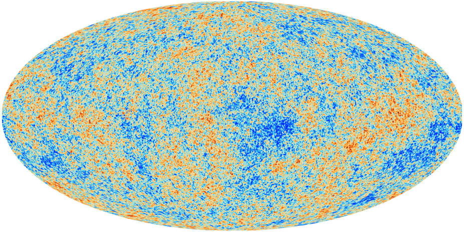
I am going to be very modest here

Put at use the fact that Nature shows a tendency toward being liberated from scales

See what this implies for phenomenology if taken at face value

Possible relevance of scale or conformal invariance

• Almost flat, scale-invariant, CMB spectrum



• The SM at the classical level contains only one dimensionful parameter, the Higgs mass M_H (in the absence of gravity).

Scale- & conformally- invariant for $M_H = 0$

Possible relevance of scale or conformal invariance

Could it be that CFTs play a fundamental role in Nature?

When this symmetry is exact it has some “peculiar” implications:

- Forbids the presence of dimensionful parameters
- No particle interpretation—the spectrum is continuous

But Nature (SM) has:

- dimensionful parameters
- particles

The central role of gravity

In one way or another, the symmetry needs to be broken for the picture I am trying to paint be phenomenologically acceptable.

In addition, gravity has to enter the game for this picture to be complete.

The mere presence of gravity necessarily breaks the symmetry.

The central role of gravity

Gravity-induced conformal symmetry breaking may be effectuated in:

A. maximally brute-force manner, i.e. couple conformal SM to gravity such that scale (& conformal) transformations are broken

B. maximally "consistent" with the symmetry manner, i.e. couple conformal SM to gravity in a scale- or Weyl- invariant manner *

* When talking about gravity we have to be careful and differentiate between conformal and Weyl ($\equiv$ gauged dilatations) [Karananas, Monin '15]

A. An almost scale-invariant Universe

Constructing the action

Selection rules: [Karananas, Shaposhnikov, Shkerin, Zell '21]

- The purely gravitational part of the action contains operators of mass dimension not greater than 2 $\leftrightarrow$ only massless graviton in the gravity spectrum
- The matter Lagrangian comprises the SM with $M_{\mathrm{Higgs}} = 0$.
- The coupling of matter to gravity only happens through operators of mass dimension not greater than 4 $\leftrightarrow$ "logical" to impose, but may be relaxed

Constructing the action

Naively simple

$$S \sim \int (M^2 + \xi h^2) R - \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4} h^4 + \dots \ ,$$

with $R = \text{Ricci scalar(metric)}$, $h =$ Higgs field in unitary gauge and ... stand for the rest of the SM.

For $M^2 \ll \xi h^2$, nontrivial modification to the dynamics[1]

Nonminimal coupling is actually a kinetic mixing between Higgs & graviton operative at high-energies relevant in the early Universe → Higgs inflation [Bezrukov, Shaposhnikov '07]

[1] For $M^2 \gg \xi h^2$, standard SM & gravity, but range of validity lowered to M_P/ξ instead of M_P

The success of Higgs inflation is inevitable

Untangle the "mess" by bringing gravity to its usual, Einstein-Hilbert, form (via Weyl rescaling)

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2}\frac{M_P^2}{\kappa}\frac{(\partial_\mu h)^2}{h^2} - \frac{\lambda M_P^4}{4\xi^2}\left(1 - \frac{\#}{h^2} + \dots\right)$$

Highly suggestive form - canonicalize via exponential map

$$h = M_P \mathrm{e}^{\sqrt{\kappa}\chi/M_P} \ , \quad \kappa = \frac{\xi}{1+6\xi}$$

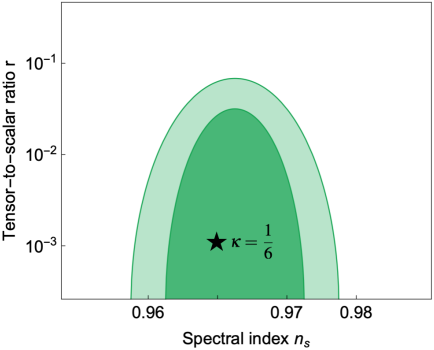
In terms of χ

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{\lambda M_P^4}{4\xi^2}\left(1 - 2\mathrm{e}^{-\sqrt{\kappa}\chi/M_P} + \dots\right)$$

Deviation from exact de Sitter is exponentially small

The success of Higgs inflation is inevitable

approximate scale symmetry, broken spontaneously $\rightarrow$ Higgs is the pseudo Nambu-Goldstone boson $\rightarrow$ approximate shift symmetry $\rightarrow$ exponentially flat potential $\rightarrow$ excellent agreement with observations

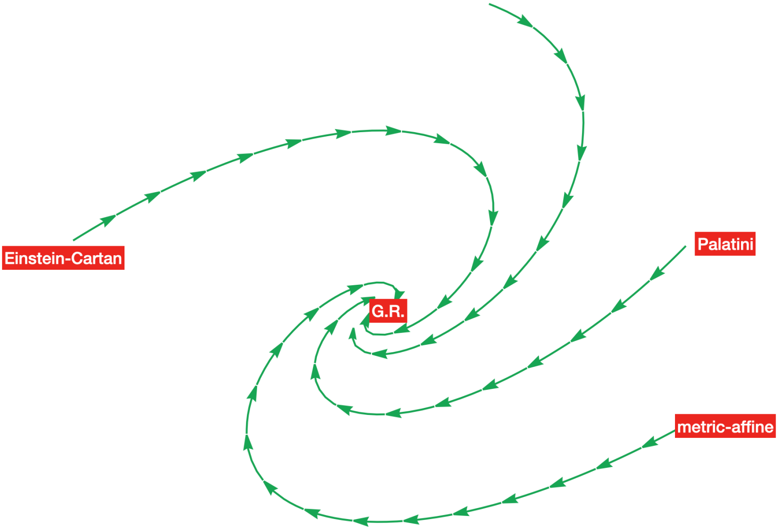


CMB normalization $\rightarrow \xi \sim 10^4$

amount of gravitational waves $r(\kappa) \sim \frac{10^{-4}}{\kappa} \quad \kappa = \frac{\xi}{1+6\xi} = \frac{1}{6}$

The non-uniqueness (?) of Higgs inflation
Rethinking GRavity

But, *which* gravity?
or, better ask *which formulation* of gravity?



G.R. (metrical gravity) is the attractor in this "landscape" of formulations

But, *which* gravity?

or, better ask *which formulation* of gravity?

- purely metrical
- Palatini
- Einstein-Cartan (EC)
- metric-affine

Does it really matter? e.g. [Shaposhnikov, Shkerin, Zell '20 & Karananas, Shaposhnikov, Shkerin, Zell '21]

- Only massless graviton & absence of matter: the above *completely equivalent*
- Only massless graviton & presence of matter: the above *not equivalent anymore*

Einstein-Cartan(-Sciama-Kibble) theory

In what follows I'll only discuss EC gravity $\rightarrow$ interaction follows from the gauge principle $\rightarrow$ as close as it gets to particle physics

Gauge shifts & Lorentz transformations, by introducing the tetrad e and connection ω with their corresponding field strengths

$$\text{torsion:} \quad T \sim \partial e + \omega e$$
$$\text{curvature:} \quad F \sim \partial \omega + \omega^2$$

Pure EC gravity

The aim is to construct a gravitational sector that propagates only a massless spin-2 field

Start by writing all possible terms—there are ten of them—up to two derivatives of the fields

Schematically:

$$S_{\rm gr} \sim \int \text{cosm. const.} + 2\times\text{curvature scalars} + 2\times\partial(\text{torsion}) + 5\times\text{torsion}^2$$

Appearances are (very) deceiving...

The connection and thus torsion is not dynamical

Pure EC gravity

Everything becomes transparent by obtaining the equivalent metric theory:

1. vary the action wrt the nondynamical connection ω
2. Solve its algebraic eom (easy)

$$\delta_\omega S_{\rm gr} = 0 \quad \leftrightarrow \quad \omega \sim \partial e$$

3. Plug the above back into the action to get

$$S_{\rm gr} \sim \int \text{cosm. const.} + \text{Ricci scalar(metric)} \ ,$$

which is nothing more than the Einstein-Hilbert action.

Logic is the same with matter, simply $\exists$ more "ingredients"

The consequences are not the same though

SM & EC gravity

The interaction of fields, specially Higgs with gravity is modified as compared to the metrical gravity [Shaposhnikov, Shkerin, Timiryasov, Zell '20 & Karananas, Shaposhnikov, Shkerin, Zell '21]

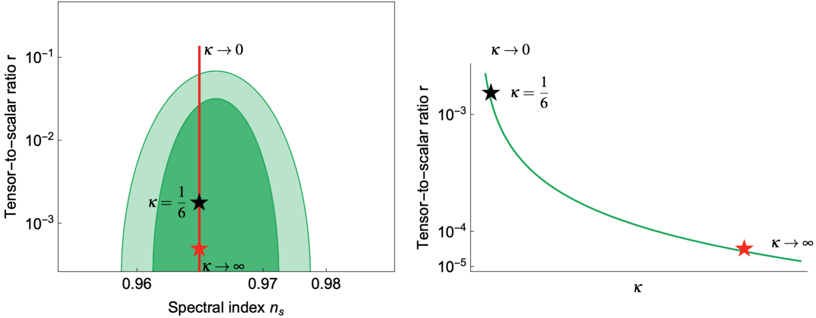
$$S \sim \int (M^2 + \xi h^2)\text{curvature scalars} \quad + \quad \zeta h^2 \; \partial(\text{torsion})$$
$$+ (M^2 + \eta h^2)\text{torsion}^2 \quad - \quad \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4} h^4 + \dots$$

At the same time, the principle underlying the inflationary dynamics is still there $\rightarrow$ approximate scale/shift symmetry

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{\lambda M_P^4}{4\xi^2}\left(1 - 2\mathrm{e}^{-\sqrt{\kappa}\chi/M_P} + \dots\right) , \quad \kappa = \kappa(\xi, \zeta, \eta)$$

The slope of the potential is controlled by κ, and so does the production of gravitational waves

Higgs inflation in EC gravity



B. A scale-invariant Universe

Why bother?

So far I discussed conformality being broken in a brute-force manner, but most statements are applicable to the symmetry-consistent manner too

Now I will focus on the situation where gravity enters the picture in a scale- or Weyl- invariant manner

This is where things become even more interesting

Scale invariance + Conformal SM + gravity

The need for an additional dilaton

First of all, the action must be liberated from any explicit scales, i.e. $M_H = 0$ and $M_P = 0$; in other words, my starting point is

$$S \sim \int \xi h^2 \text{curvature scalar} \quad + \quad \zeta \partial h^2 \text{ torsion}$$
$$+ \eta h^2 \text{torsion}^2 \quad - \quad \frac{1}{2}(\partial_\mu h)^2 - \frac{\lambda}{4} h^4 + y h \bar{\psi}\psi \ + \ldots$$

Unsatisfactory for particle physics & cosmological phenomenology, or how I managed to ruin SM & early Universe in one try! ☺

Literally, it's the EC version of induced gravity

Scale invariance + SM + gravity
The need for an additional dilaton

Untangle the "mess" by finding the equivalent metric theory

- Integrate out the connection (still nondynamical!); in particular

$$\omega \sim \partial e + \partial h + \dots$$

- Go to Einstein frame by Weyl rescaling the metric

The result is

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2}(\partial_\mu \chi)^2 - \frac{\lambda M_P^4}{4\xi_h^2} + y \frac{M_P}{\sqrt{\xi_h}} \bar{\psi}\psi + \dots \ , \quad \chi = M_P e^{\frac{h}{M_P}}$$

Minimally coupled massless scalar field interacting with matter derivatively and gravitationally...

The origin of scales

A viable scale-invariant embedding of the conformal SM requires the introduction of a massless scalar field, the dilaton

This is the scale donor: the Planck mass is generated dynamically via spontaneous symmetry breaking

$$< \mathrm{dilaton} > \to \; M_{\mathrm{Planck}} \; \to < \mathrm{Higgs} >$$

The dilaton may play interesting role in the late Universe, being responsible for the present-day accelerated expansion

Vanishing vacuum energy in SSB CFTs

The vacuum energy in such constructions is automatically zero, in spite of the fact that scales have been generated

Nontrivial statement, but it literally follows from dimensional analysis: the potential is a homogeneous function of the fields, or in other words

$$V \propto \phi \frac{\partial V}{\partial \phi}$$

SSB means $< \phi > \neq 0$, thus

$$\left. \frac{\partial V}{\partial \phi} \right|_{<\phi>} = 0 \quad \rightarrow \quad V(< \phi >) = 0$$

Inflationary dynamics = practically single-field

Predictions in a wide class of models appear universal and independent of the details, effectively Higgs inflation [Karananas, Rubio '16 & Karananas, Michel, Rubio '20 & Karananas, Shaposhnikov, Zell '23]

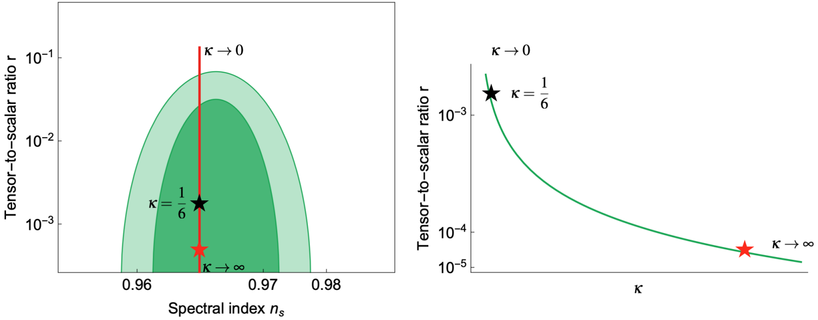
The kinetic terms of the scalars span a two-dimensional manifold

$$S \sim \int \frac{M_P^2}{2} R - \frac{1}{2} \begin{pmatrix} \partial\chi & \partial h \end{pmatrix} \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{12} & \gamma_{22} \end{pmatrix} \begin{pmatrix} \partial\chi \\ \partial h \end{pmatrix} - V(\chi, h) \ ,$$

with $\gamma_{ij} = \gamma_{ij}(\chi, h)$

Conformal symmetry broken by gravity fixes the curvature of this manifold to be constant—coincides with κ

Inflationary dynamics = practically single-field



Intricate link between "geometry" & observables

Weyl invariance

One may feel uneasy that we introduced another scalar (although this can play an interesting role for late Universe evolution)

One may feel even more uneasy working with an action that contains that many terms (despite the universality)

Weyl invariance as a "book-keeping device": reduce the number of admissible terms and also rid of the dilaton - spurious field

The graviscalar sector propagates the massless graviton and one scalar, only

Weyl invariance

Rare situation that gauging a symmetry doesn't bring in new degrees of freedom

Aftermath of the fact that geometrical data transform nontrivially under Weyl rescalings - they pick up inhomogeneous pieces involving derivatives of $q(x)$

$$g_{\mu\nu} \to \hat{g}_{\mu\nu} = q^{-2}(x) g_{\mu\nu} \ , \quad R \to \hat{R} = R + \partial q \ .$$

They play the role of the corresponding gauge field

Weyl invariance and metrical gravity

Well known example, conformally coupled scalar φ in metric gravity

$$S = \int d^4x\sqrt{g}\left(\frac{\varphi^2}{12}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi - \frac{\lambda}{4}\varphi^4\right)$$

One can have Weyl invariance without ad hoc introduction of new degrees of freedom, while φ is artifact

Weyl invariance and EC gravity

In EC gravity the situation is different and interesting

Curvature F is inert, but the trace of torsion, say v_μ, transforms inhomogeneously

$$v_\mu \mapsto v_\mu + 3q^{-1}\partial_\mu q \ ,$$

meaning that $\frac{v_\mu}{3}$ is the Weyl vector in disguise

Interestingly, (very) similar considerations apply if instead of torsion one considers non-metricity [Ghilencea '19-'22]

Inflationary dynamics = genuinely single-field

The graviscalar Weyl-invariant incarnation of Higgs-dilaton in the Einstein frame [Karananas, Shaposhnikov, Shkerin, Zell '21]

$$S=\int d^4x\sqrt{g}\left[\frac{M_P^2}{2}R-\frac{K(h)}{2}(\partial_\mu h)^2-\frac{\lambda M_P^4}{4}\frac{h^4}{(M^2+\xi h^2)^2}\right]\ .$$

More constrained, but with enough freedom to get satisfactory phenomenology

E.g. take the Weyl-Palatini model, where only (nonminimal) interactions with the scalar curvature are kept; at field values relevant for inflation

$$K(h)\sim\frac{M_P^2}{\xi h^2}\ ,$$

yielding the nice, exponentially-flat plateau we saw before

Recap

- Conformal symmetry may be the key for the harmonic symbiosis of gravity & SM
- If inflation is the mechanism responsible for the isotropy and homogeneity of our Universe, then the Higgs field is responsible for inflation
- Gravity plays an important role, being responsible for breaking conformality
- Einstein-Cartan gravity is as close as it gets to the particle physicist's mindset
- Combining gravity & SM in a scale- or Weyl- invariant manner offers(?) insights on the cosmological constant problem

Thank you!