Renormalisation of essential operators in Asymptotic Safety - a status report

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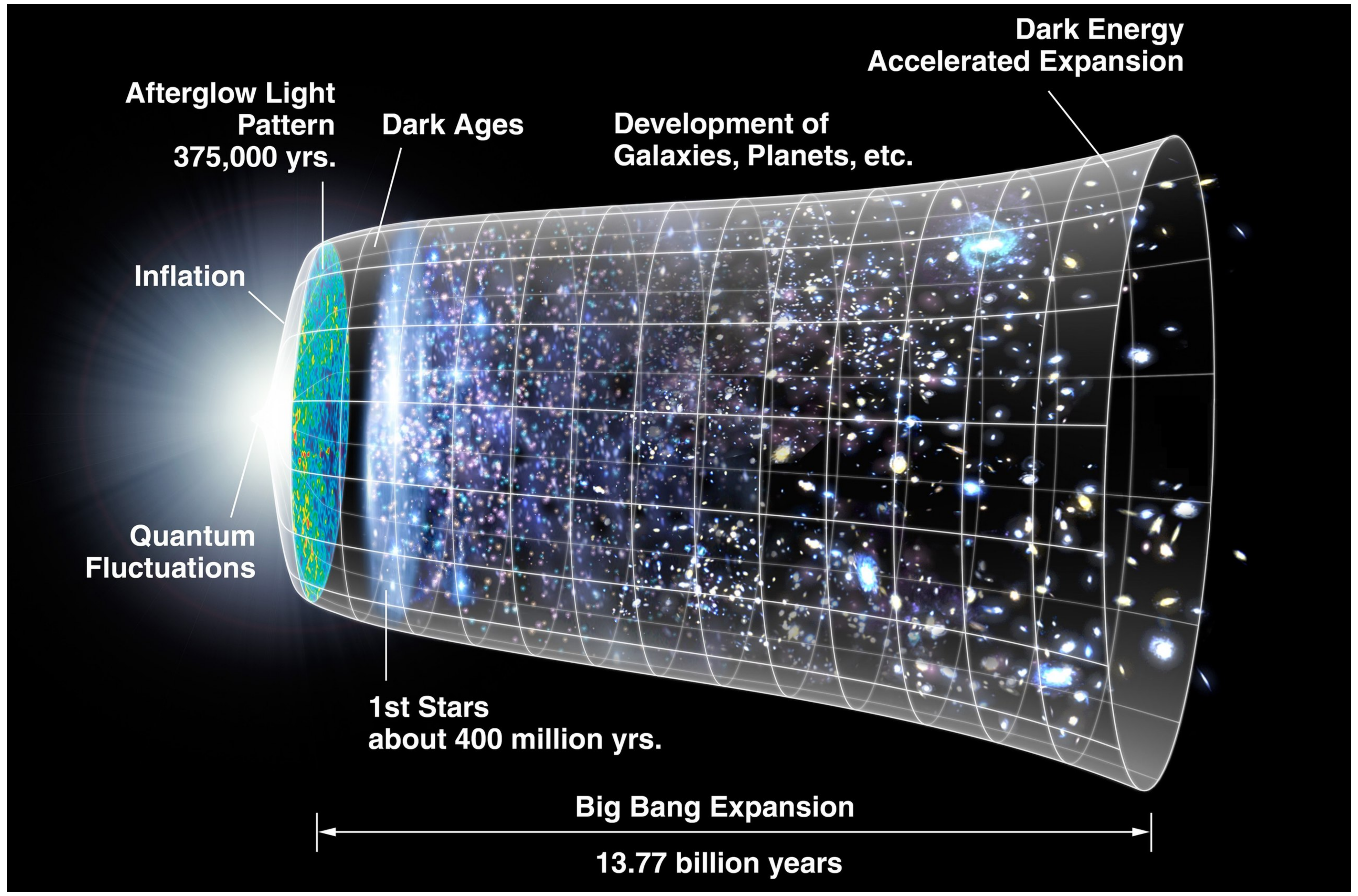
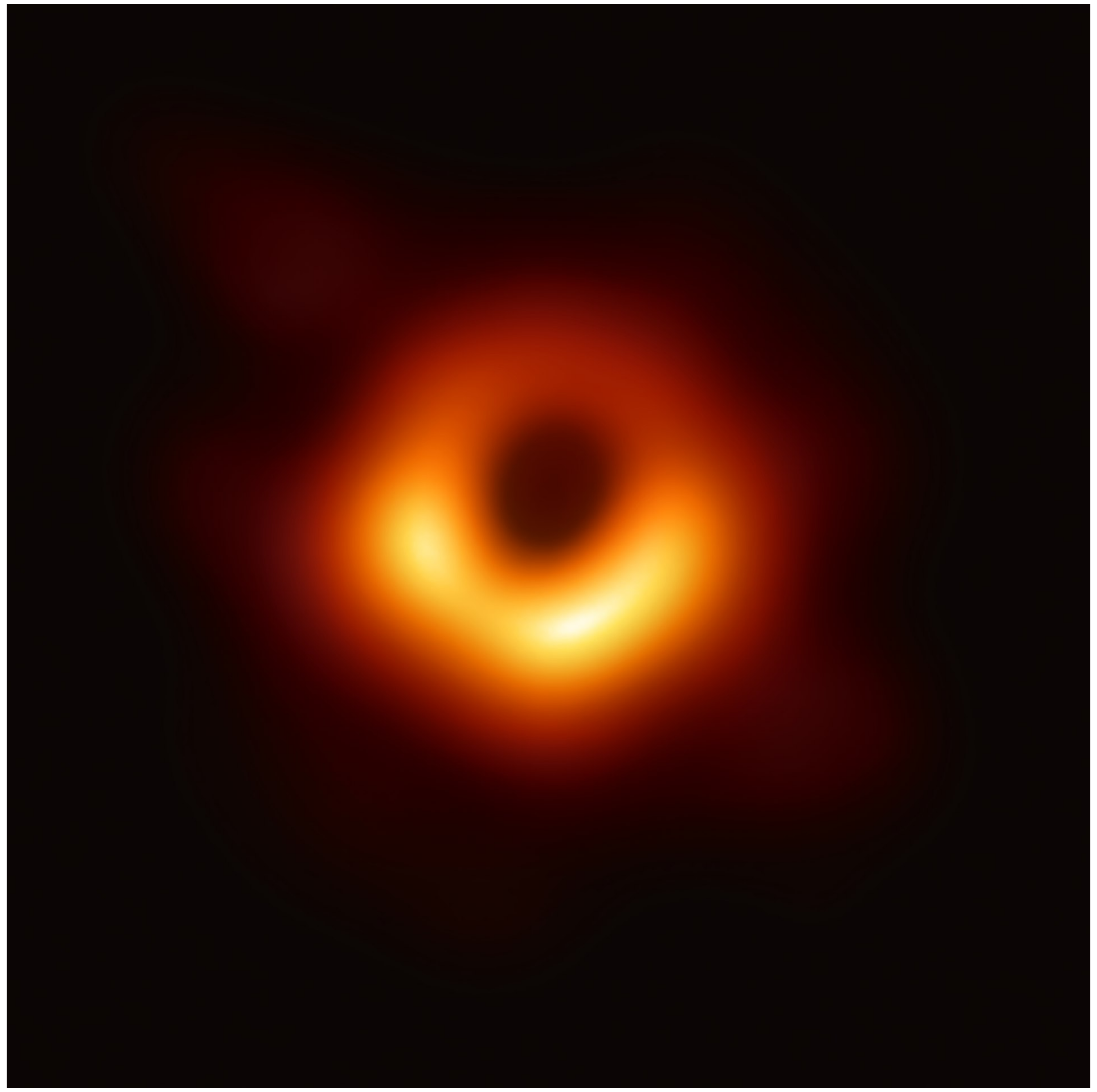
A Perspective on Quantum Gravity





A Perspective on Quantum Gravity

lack of smoking gun
quantum gravity experiments

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many big ideas
and even bigger claims
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**why trust any
approach in particular?**

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    1. set up quantum theory of gravity and matter (at least SM)

    2. confront the theory with as much available theory constraints (unitarity, causality, …) and experimental data (cosmological evolution, particle masses, …) as possible

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- less glamorous, more down-to-earth: fix starting point and see how far you get

A Perspective on Quantum Gravity

- working hypothesis: QFT works also for QG
    - access to standard QFT notions like **renormalisation group** and **scattering amplitudes**
- in this talk: work towards $(2 \to 2)$ amplitudes in Asymptotic Safety, but general ideas can be carried over to other approaches

Outline

- Scattering amplitudes, field redefinitions and (in-)essential couplings
- Essential couplings in quantum gravity
- RG of essential QG - selected results
- Summary

Scattering amplitudes,
field redefinitions and (in-)essential couplings

Scattering amplitudes

- within Asymptotic Safety: most investigations at level of off-shell RG flow
  - residual dependencies on parameterisation, gauge choice etc.
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read fine print for details

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inessential couplings
multiply EoM in action

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Effective Action

- for now: talk about **exact** theory (later: approximations) in the "not too strongly" interacting regime
  - hierarchy in the importance of correlation functions
  - on-shell expansion central to make sense of this
- recall: effective action includes all quantum effects, full scattering amplitudes obtained from tree level diagrams

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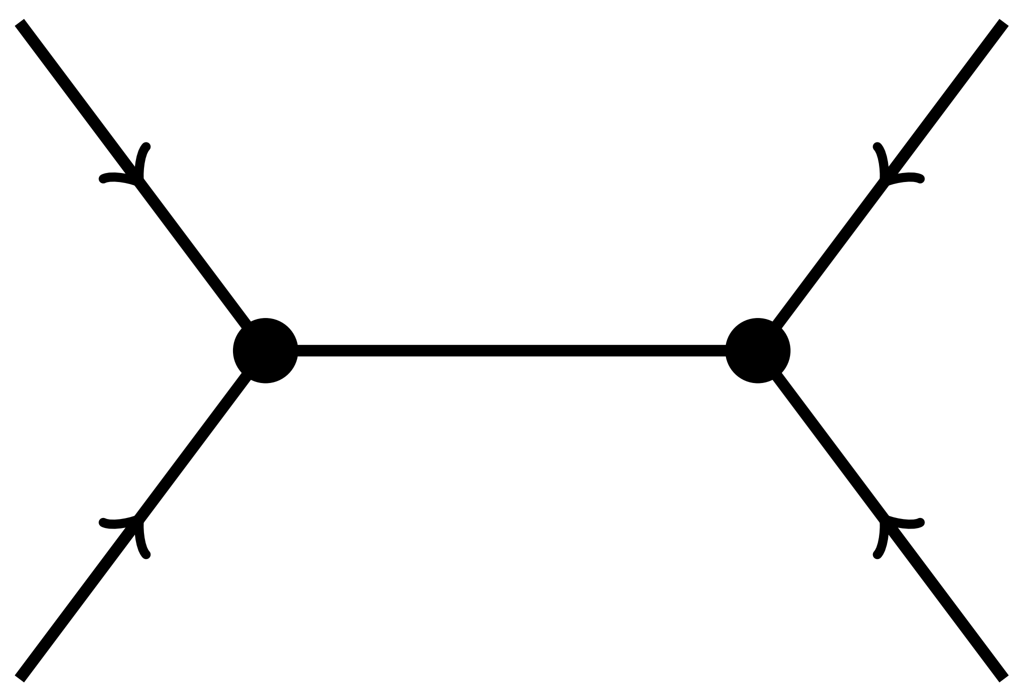

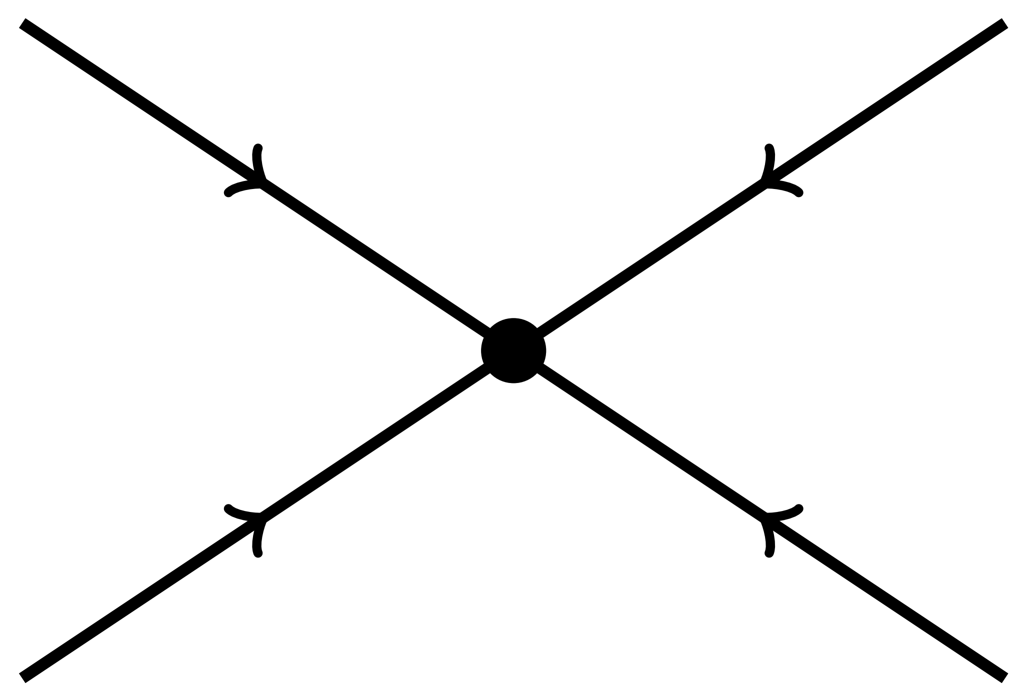

Field redefinitions - some examples

- consider "standard" scalar field theory in Minkowski space
  - wave function renormalisation inessential: $\phi \mapsto Z(p^2)^{-1/2}\phi$
  - removes non-trivial momentum dependence of propagator by shifting it into interactions
  - caveat: needs $Z > 0$, cannot remove or add modes
  - what about **branch cuts** (e.g. 1-loop logs)?

Field redefinitions - some examples

- infinite derivative scalar field theory in Minkowski space:

$$\Gamma = \int \mathrm{d}^d x \, \frac{1}{2} \phi \frac{\Box^2 + (m_1^2 + m_2^2)\Box + m_1^2 m_2^2}{2\Box + m_1^2 + m_2^2} \phi + \mathcal{O}(\phi^4)$$

Field redefinitions - some examples

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$\Rightarrow$ two non-ghostly modes

Field redefinitions - some examples

- consider Starobinsky-type theory
  - spectrum: EH + massive scalar
  - field redefinition can change value of R^2 coupling, but cannot be set to zero (removes mode)

Lessons learned

- in practice: have to **choose universality class** that we are investigating beforehand
    - adapt regularisation, make specific choices for inessential couplings to simplify computations
    - hierarchy - expansion about on-shell configuration
    - how to treat branch cuts, (strong) IR non-localities, bound states, …?

Lessons learned

- in practice: have to **choose universality class** that we are investigating beforehand

- interpretation of approximation:

  - choose UC $\rightarrow$ adapt field redefinition to remove inessential operators $\rightarrow$ approximate

  - conceptually different from first approximating (introduces extra modes), then field redefinition (potentially removing extra modes)

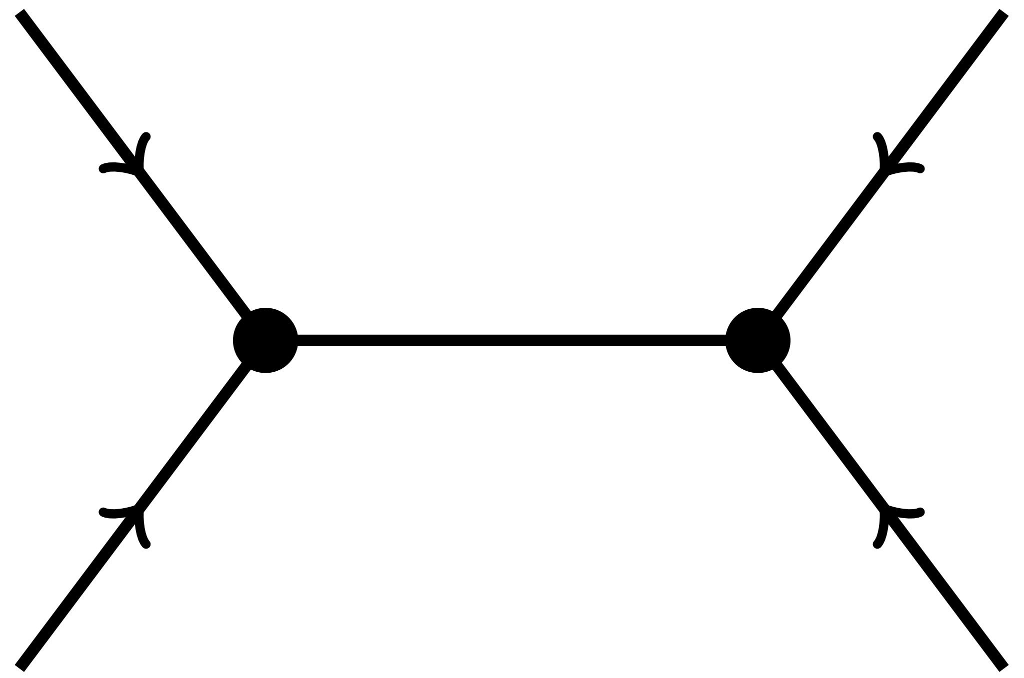
especially relevant in derivative expansions: *Platania, Wetterich '20*
Platania '22

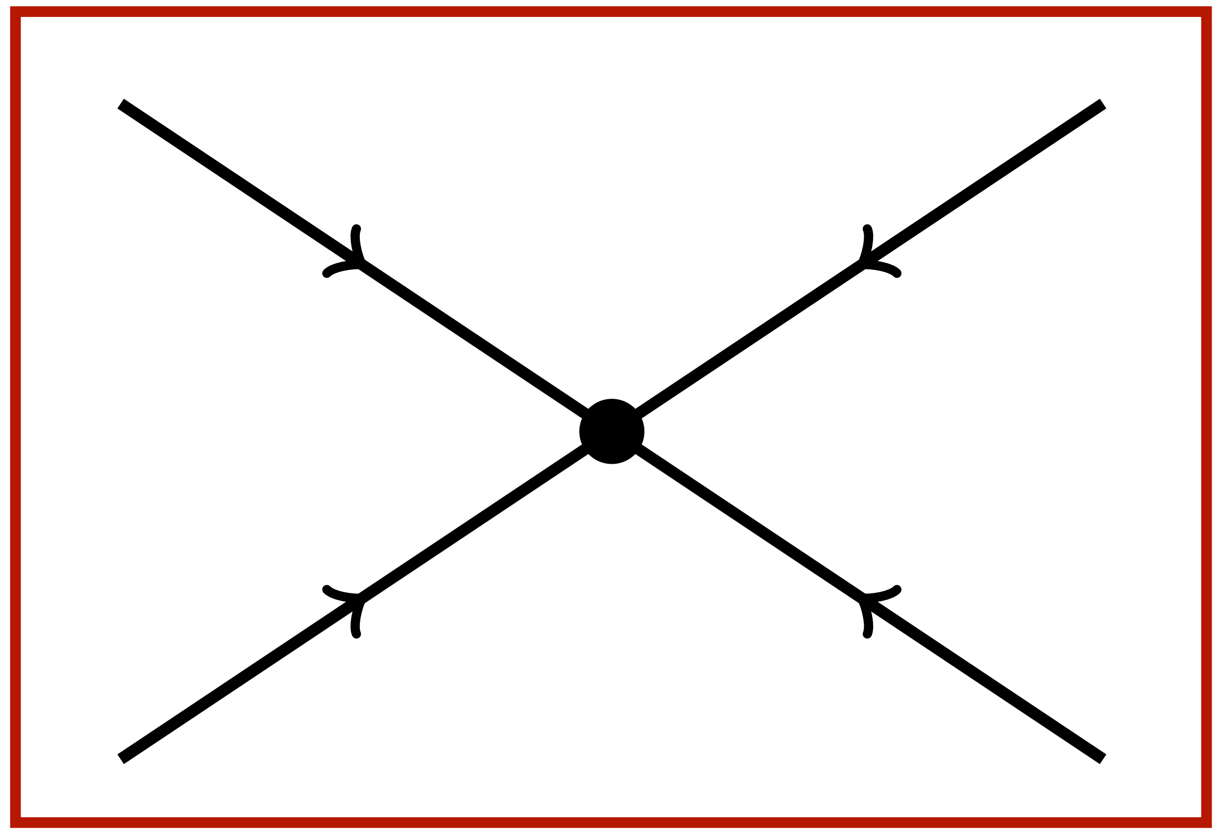
Lessons learned

- in practice: have to **choose universality class** that we are investigating beforehand
- interpretation of approximation
- in the following: discuss **[GR]** (+matter in **[Gauss]**)
    - only “standard” modes (no extra modes, ghostly or not)

Lessons learned

- in practice: have to **choose universality class** that we are investigating beforehand
- interpretation of approximation
- in the following: discuss **[GR]** (+matter in **[Gauss]**)
- philosophy: any inessential couplings that can be set to zero should be set to zero (“minimal essential scheme”)





Essential couplings in quantum gravity

Essential couplings in **[GR]**

Field redefinitions in QG

- general field redefinition of the metric:

$$g_{\mu\nu} \mapsto c_0 g_{\mu\nu} + c_1(\Box) R_{\mu\nu} + c_2(\Box) R \, g_{\mu\nu} + \dots$$

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(is this still a Lorentzian metric?)

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- curvature expansion of effective action of **[GR]** in minimal scheme:

$$\Gamma_{[\mathbf{GR}]} = \frac{1}{16\pi G_N} \int \mathrm{d}^d x \, \left[2\Lambda - R + \Theta \mathfrak{E} + G_{C^3} C^3 + \mathcal{O}(\mathcal{R}^4) \right]$$

NB: only one of G_N, Λ essential,
cannot set either to zero generically

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- quadratic terms inessential: multiply EoM

$$\left(R - \frac{2d}{d-2}\Lambda + \dots\right) f_R(\Box) \left(R - \frac{2d}{d-2}\Lambda + \dots\right)$$

$$\left(S_{\mu\nu} + \dots\right) f_S(\Box) \left(S^{\mu\nu} + \dots\right)$$

$$\Box C = \{DDS, DDR\}$$

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- cubic terms almost all inessential, except Goroff-Sagnotti term

no essential $D^2C^3, \ldots, D^{12}C^3$ terms
(Bianchi + partial integration)

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structurally, $f(s,t)\, C^4$

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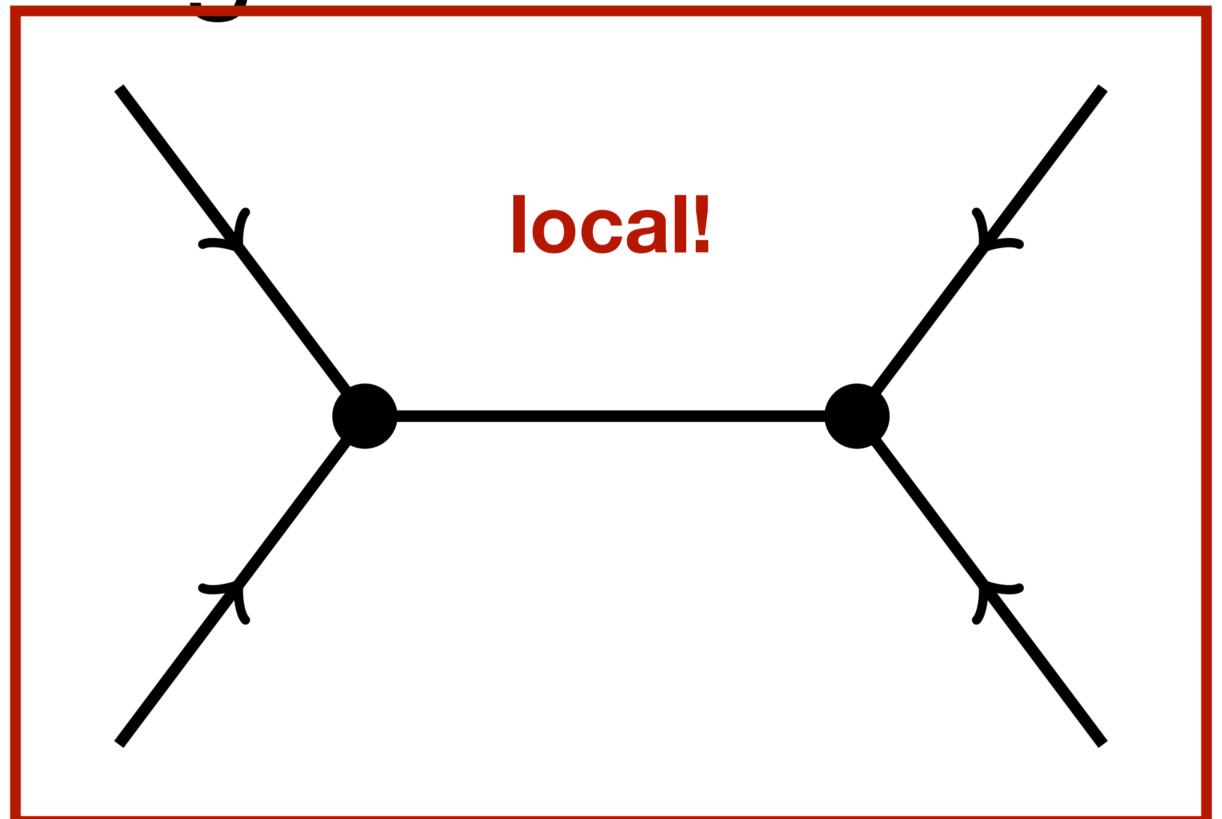

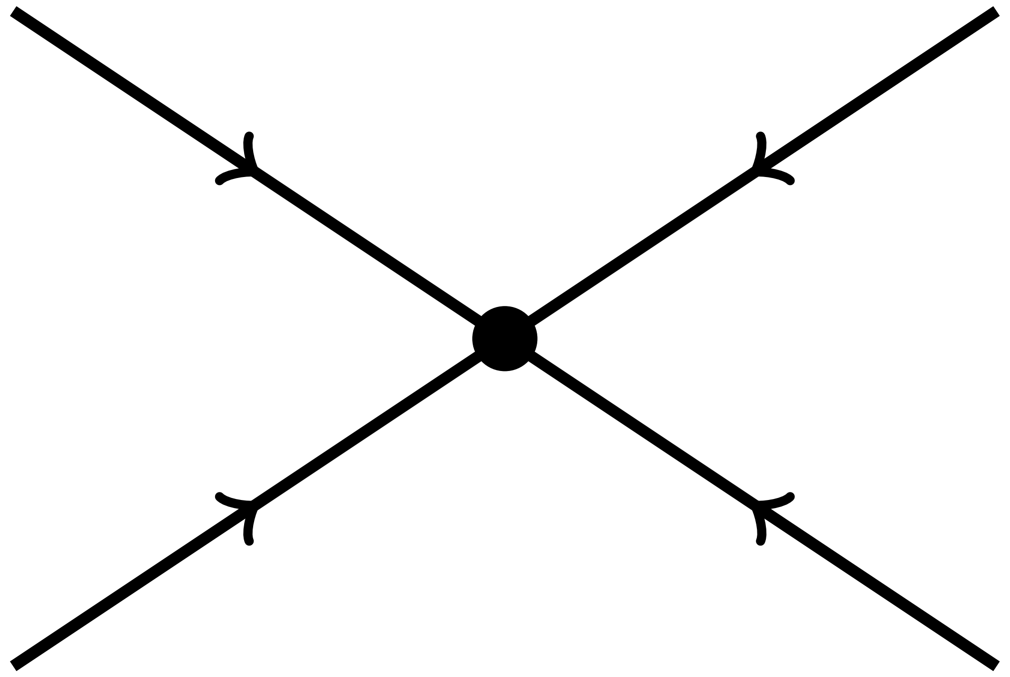

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- in practice: have to impose field redefinition consistently along RG flow - reduction of complexity, but no free lunch

- non-minimal coupling to matter: only involves Weyl tensor → expected to be extremely suppressed

$$C^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

RG of essential QG
selected results

FRG equation in essential scheme:
Baldazzi, Ben Alì Zinati, Falls ’21
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Baldazzi, Falls, Kluth, BK (WIP)

RG flow in [GR]

- consider derivative expansion up to 6th order (d=4):

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10 running field redefinitions!

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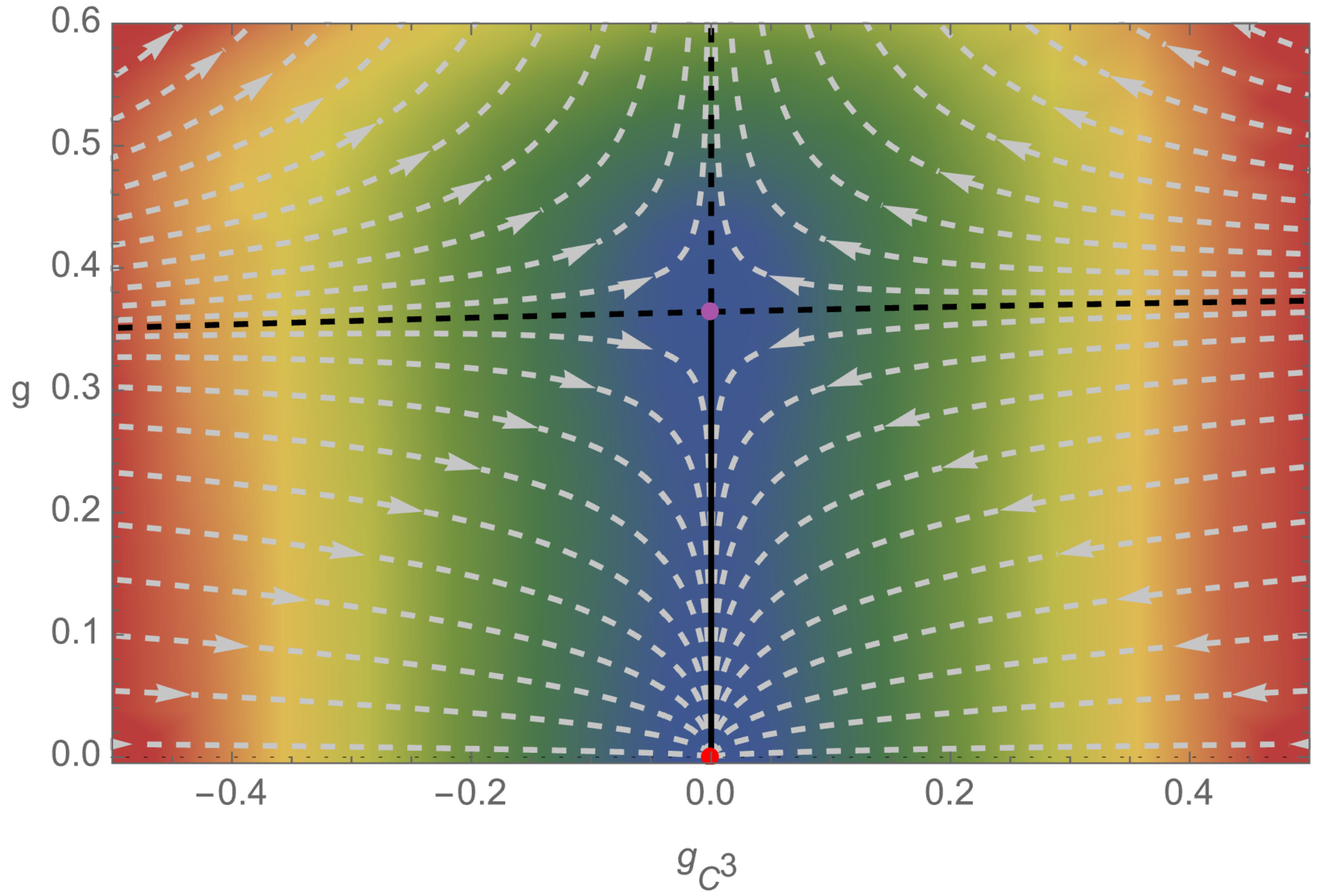

$$\theta_1 = 2.225$$

$$\theta_2 = -3.850$$

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$$G_{C^3} = \left(\frac{G_N}{16\pi}\right)^2 \left(a - \frac{86}{315}\ln G_N k^2\right)$$

$$a = -0.5065$$

RG flow in **[GR]**

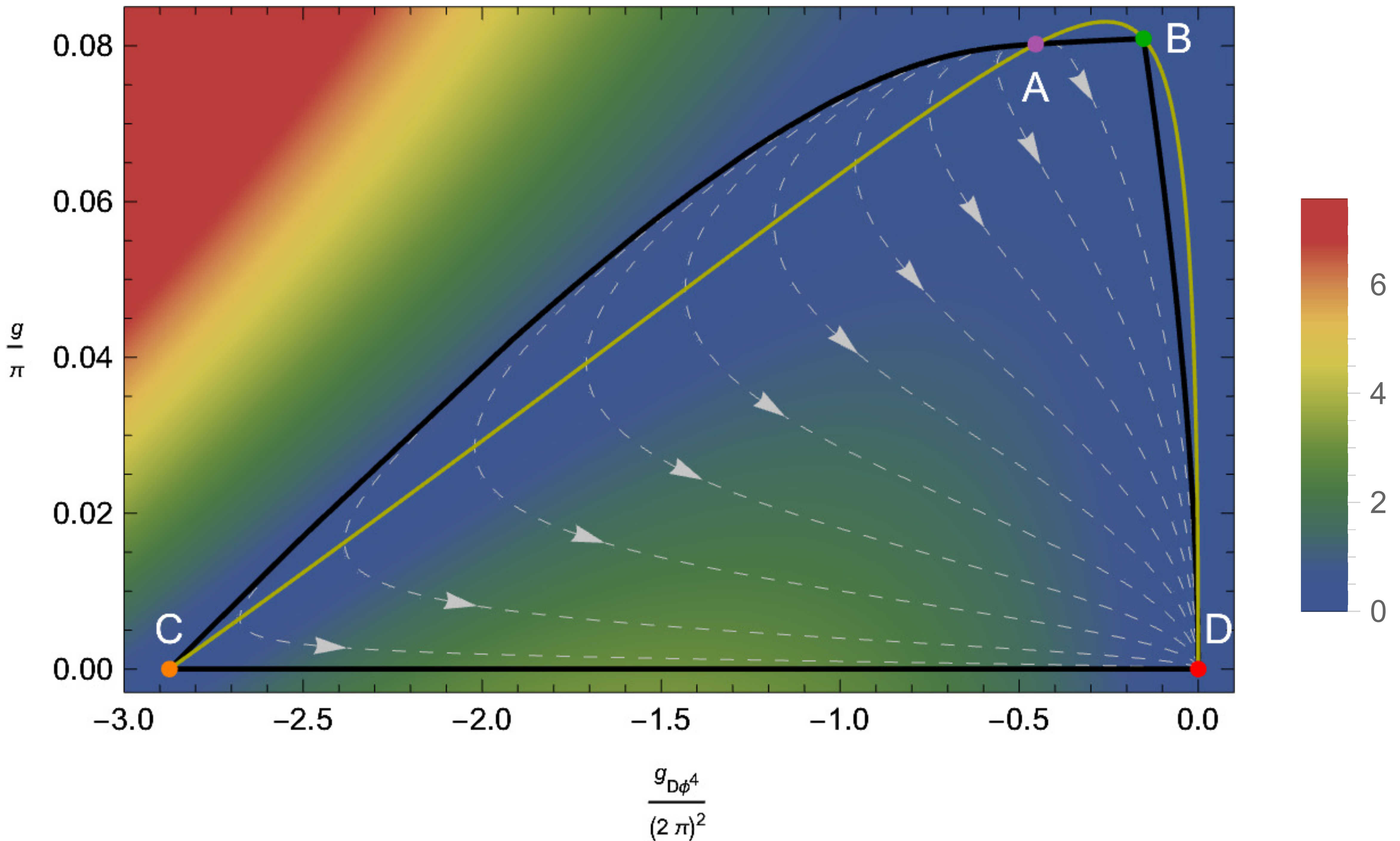
- work in progress:
  - curvature expansion up to second order (two momentum-dependent running field redefinitions) *BK, Ripken*
  - "f(R)" in **[GR]** *BK, Sannestedt*
  - general dimension, gauge dependence, on-shell flow

RG flow in [GR]+[Gauss]

- consider derivative expansion up to 4th order coupled to matter:

$$\Gamma_{[\mathbf{GR}]+[\phi]} = \int \mathrm{d}^4x \left(\frac{1}{16\pi G_N} \left[2\Lambda - R + \Theta \mathfrak{E}\right] + \frac{1}{2}(D_\mu \phi)^2 + G_{D\phi^4} \left(\frac{1}{2}(D_\mu\phi)^2 \right)^2 \right)$$

BK '22



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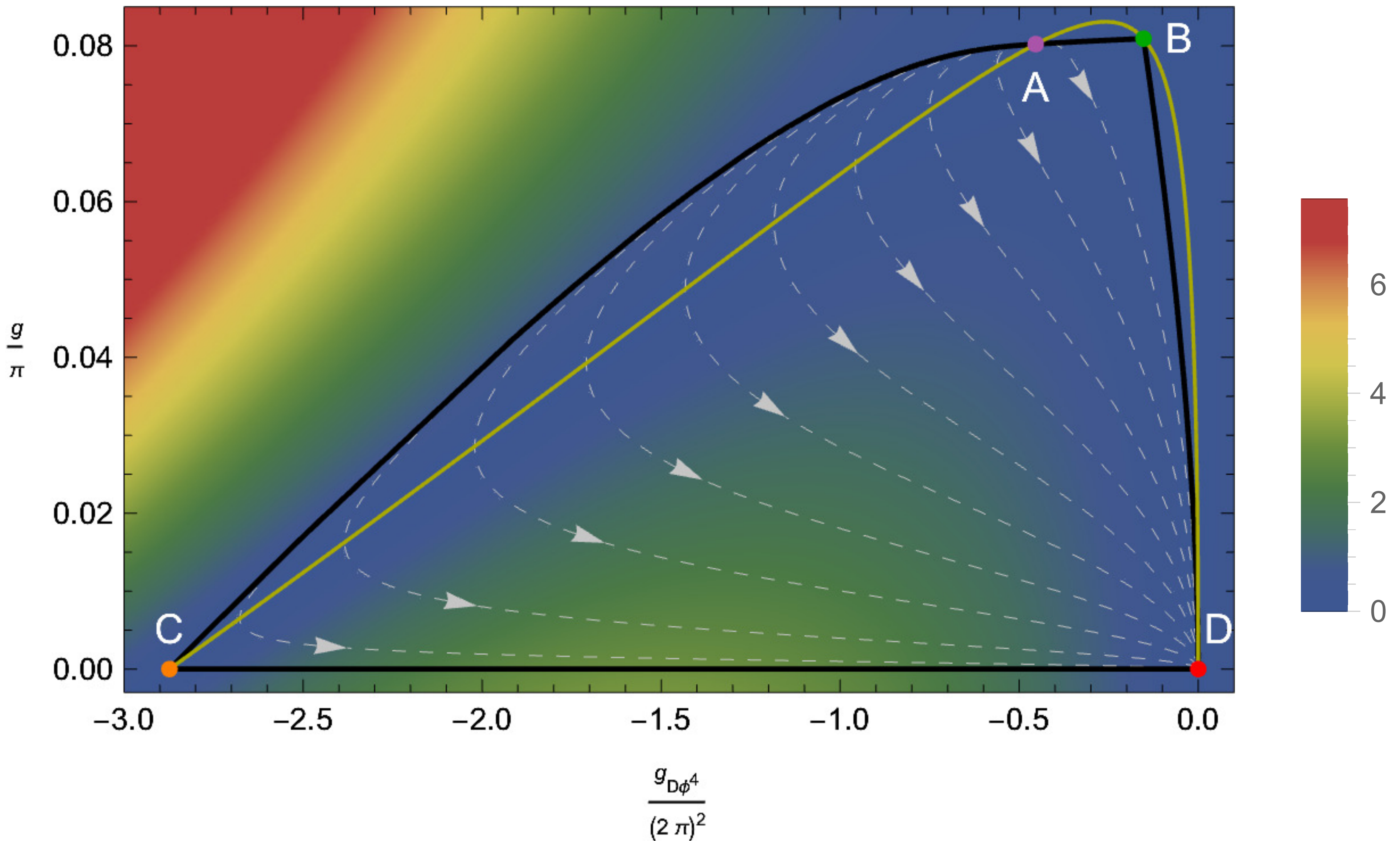

A+C potentially fake,
B+D stable
[*de Brito, BK, Schiffer '23*]

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BK '22

$$\Gamma_{[\mathbf{GR}]+[\gamma]} = \int \mathrm{d}^4x \left(\frac{1}{16\pi G_N} \left[2\Lambda - R + \Theta\mathfrak{E}\right] + \frac{1}{4}\mathrm{tr}F^2 + G_{\mathcal{F}^2} \left(\frac{1}{4}\mathrm{tr}F^2\right)^2 + \frac{G_{F^4}}{16}\mathrm{tr}F^4 + G_{CFF}C^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} \right)$$

BK, Platania (WIP)

very similar phase diagram
potentially intriguing relation to positivity/weak-gravity bounds

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many more systems under investigation
stay tuned!

Summary

Summary

- falsifiability is at the heart of science, and it should also be at the heart of quantum gravity research

- scattering amplitudes are promising tool to probe quantum gravity

- field redefinitions allowed - all the difficulty starts at four-point function, strong results need high level of sophistication

- **[GR]** promising from perspective of Asymptotic Safety - maybe no free parameters? AS **[GR]** = string theorists' dreams come true?