Classical and Quantum cosmology

of the (very) late Universe.

Quantum Gravity and Cosmology (2023 conference)

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Table of contents

Introduction

Introduction-1-: A brief sketch of the universe

- The universe is homogeneous and isotropic on large scales (cosmological principle)
- The matter content of the universe:
    - Standard matter
    - Dark matter
    - Something that induce the late-time acceleration of the Universe
- The acceleration of the universe is backed by several measurments: $H(z)$, SneIa, BAO, CMB, LSS (matter power spectrum, growth function)...

Introduction-2-

- The effective equation of state of whatever is driving the current speed up of the universe is roughly -1. For example, for a wCDM model with w constant and $k = 0$, Planck (TT, TE, EE+lensing) + ext(BAO,H0,JLA) results implies w is very close to -1
- Such an acceleration could be due
    - A new component of the energy budget of the universe: dark energy; i.e. it could be Λ, quintessence or of a phantom(-like/effective) nature
    - A change on the behaviour of gravity on the largest scale. No new component on the budget of the universe but rather simply GR modifies its behaviour, within a metric, Palatini (affine metric) ....

Late-time acceleration of the Universe within GR: dark energy with a constant EoS

Constant equation of state for DE: background-1-

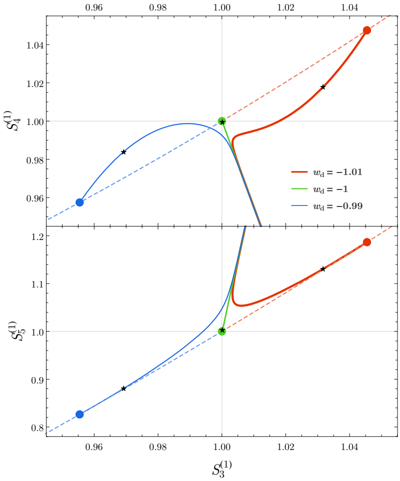
- Cosmic acceleration:

$$\frac{\ddot{a}}{a} = -\frac{4\pi \mathrm{G}}{3}(\rho_{\mathrm{m}} + \rho_{\mathrm{de}} + 3p_{\mathrm{de}})$$

- Observation indicates that for $w_{\mathrm{de}} \sim -1$ where $w_{\mathrm{de}} = p_{\mathrm{de}}/\rho_{\mathrm{de}}$.
- Therefore, as soon as DE starts dominating the Universe starts accelerating, i.e. $\ddot{a} > 0$.
- Simplest cases ΛCDM or wCDM.

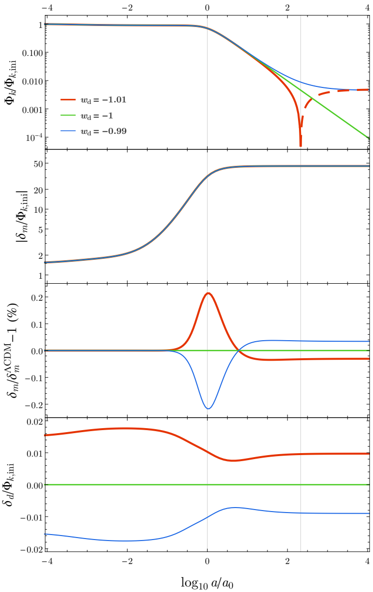
Constant equation of state for DE: background-2-

- State finders approach (Sahni, Saini and Starobinsky JETP Lett. [arXiv:astro-ph/0201498])
  - Scale factor: $\frac{a(t)}{a_0} = 1 + \sum_{n=1}^{\infty} \frac{A_n(t_0)}{n!} \left[H_0 \left(t - t_0\right)\right]^n$, where $A_n := a^{(n)}/(a\,H^n)$, $n \in \mathbb{N}$.
  - State finders parameters:
    $S_3^{(1)} = A_3$,
    $S_4^{(1)} = A_4 + 3\,(1 - A_2)$,
    $S_5^{(1)} = A_5 - 2\,(4 - 3A_2)\,(1 - A_2)$
  - $\Omega_{\rm m} = 0.309$, $\Omega_{\rm d} = 0.691$ and $H_0 = 67.74$ km s^{-1} Mpc^{-1} (according to Planck).



Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-1-



- Example of the evolution of the perturbations: $k = 10^{-3}$ Mpc^{-1}
- ΛCDM model: Φ_k vanishes asymptotically
- Phantom model: Φ_k also evolves towards a constant in the far future but a change of sign occurs roughly at $\log_{10} a/a_0 \simeq 2.33$, corresponding to 8.84×10^{10} years in the future. A dashed line indicates negative values of Φ_k
- Quintessence model: Φ_k evolves towards a constant in the far future without changing sign

Albarran, B.L. and Morais, EPJC 2018 [arXiv:1706.01484]

Constant equation of state for DE: perturbations-2-

- What about $f\sigma_8$ for the three different DE models?

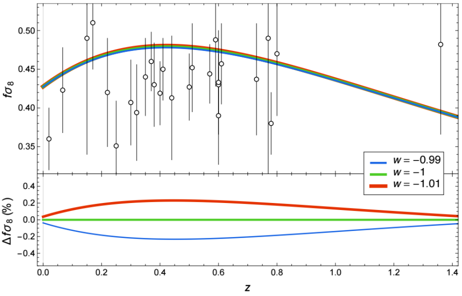


Figure 2: (Top panel) evolution of $f\sigma_8$ for low red-shift $z \in (0, 1.4)$ for three dark energy models: (blue) $w = -0.99$, (green) $w = -1$ and (red) $w = -1.01$. White circles and vertical bars indicate the available data points and corresponding error bars (cf. Table I of [13]). (Bottom panel) evolution of the relative differences of $f\sigma_8$ for each model with regard to ΛCDM ($w = -1$). $\Delta f\sigma_8$ is positive in the phantom case and negative in the quintessence case. For all the models, it was considered that σ_8 evolves linearly with δ_m and that $\sigma_8 = 0.816$ at the present time [7].

$$f \equiv \frac{d\left(\ln \delta_{\rm m}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\, k_{\sigma_8}\right) = \sigma_8\left(0,\, k_{\sigma_8}\right) \frac{\delta_{\rm m}\left(z,\, k_{\sigma_8}\right)}{\delta_{\rm m}\left(0,\, k_{\sigma_8}\right)}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\, k_{\sigma_8}) = 0.820$ (Planck)

Albarran, B.L. and Morais [arXiv:1706.01484]

Late-time acceleration of the Universe within GR and with a phantom fluid

Late-time acceleration of the Universe within GR and with a phantom fluid

The models

The models

- We are going to focus on the genuinely phantom matter. i.e. when the Equation of State satisfies $w < -1$.
- The phantom matter violates the Null energy condition. In consequence, the rest of the energy conditions are violated.
  - Null energy condition $\Rightarrow$ $p + \rho \geqslant 0$.
  - Weak energy condition $\Rightarrow p + \rho \geqslant 0$, $\rho \geqslant 0$.
  - Dominant energy condition $\Rightarrow$ $\rho \geqslant |p|$.
  - Strong energy condition $\Rightarrow$ $p + \rho \geqslant 0$, $3p + \rho \geqslant 0$.
- For example, a suitable way to write the Equation of State of a phantom fluid is

$$p = -\rho - C\rho^{\alpha},$$

where C is a positive constant and α is a real number. We are going to focus on the cases $\alpha = 1, 1/2, 0$.

Genuine phantom models: BR, LR and LSBR

- The DE content can be described for example with a perfect fluid or a scalar field

Event	EoS for a perfect fluid	Potential for a scalar field
BR	$p_d = w_{\mathrm{d}} \rho_d$	$V(\phi) = C_{br} e^{\lambda\phi}$
LR	$p_{\mathrm{d}} = -\rho - B\sqrt{\rho_{\mathrm{d}}}$	$V(\phi) = C_{lr}\phi^4 + D_{lr}\phi^2$
LSBR	$p_{\mathrm{d}} = -\rho_{\mathrm{d}} - A/3$	$V(\phi) = C_{ls}\phi^2 + D_{ls}$

Where $w_d < -1$, the parameters A and B are positive and C_{br}, C_{lr}, D_{lr}, C_{ls} and D_{lr} are constants.

- The lower is the power on ϕ of $V(\phi)$, the smoother is the abrupt event.

(1) A.A. Starobinsky. astro-ph 9912054; R.R. Cadwell astro-ph 9908168; Cadwell *et al.* astro-ph/0301273

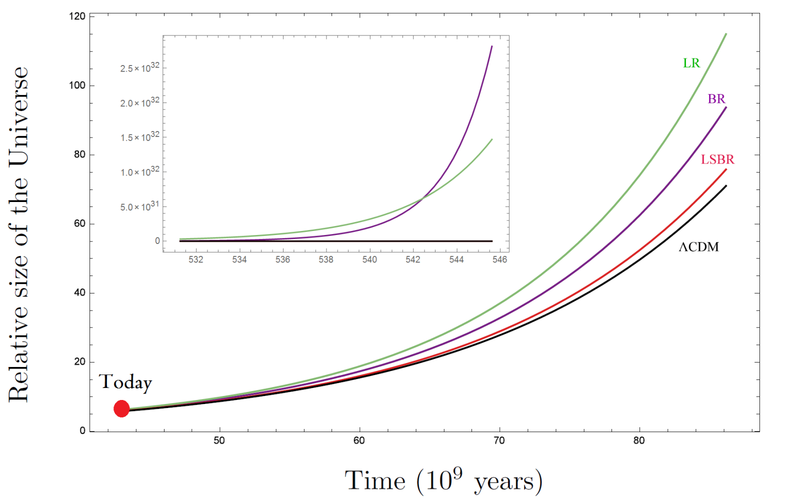
(2) H. Štefančić. astro-ph 0411630; S. Nojiri, S. Odintsov and S. Tsujikawa. hep-th/0501025

(3) M. Bouhmadi-López , A. Errahmani, P. Martín-Moruno, T. Ouali and Y. Tavakoli. arXiv:1407.2446

(4) M. P. Dąbrowski, C. Kiefer and B. Sandhöfer. hep-th/0605229

Phantom energy: Should we be afraid?

- Evolution of the scale factor for different models vs cosmic time.



Late-time singularities

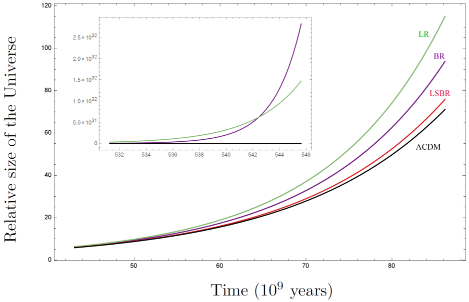
DE might induce a future cosmic singularity

Some of the cosmological parameters:

- t → Cosmic time
- a → Scale factor (relative size)
- H → Hubble parameter (growth rate)
- $\dot{H}$ → Time derivative of H

Singularity	t	a	H	$\dot{H}$	$\ddot{H}, \dddot{H}$ ...
Big Bang	0	0	∞	∞	∞
De Sitter $_{(\Lambda CDM)}$	∞	∞	H_{ds}	0	0
Big Rip	t_s	∞	∞	∞	∞
LR	∞	∞	∞	∞	∞
LSBR	∞	∞	∞	$\dot{H}_s$	0
Big Freeze	t_s	a_s	∞	∞	∞
Sudden. S.	t_s	a_s	H_s	∞	∞
Type IV	t_s	a_s	H_s	$\dot{H}_s$	∞

Asymptotic evolution of the scale factor



Bouhmadi-López, Kiefer, Martín-Moruno, arXiv:1904.01836 [gr-qc] (review published in GRG)
Borislavov Vasilev, Bouhmadi-López, Martín-Moruno, arxiv: arXiv:2106.12050 (review published in Universe)

Late-time acceleration of the Universe within GR and with a phantom fluid

Observational data and constraints

Observational data

- The Pantheon compilation: 1048 SNeIa dataset $0.01 < z < 2.26$
- The power spectrum of CMB affects crucially the physics, from the decoupling epoch till today. Effects are mainly quantified by the acoustic scale l_a and the shift parameter R Komatsu et 2008
- The BAO peaks present in the matter power spectrum can be used to determine the Hubble parameter $H(z)$ and the angular diameter distance $D_A(z)$
- $H(z)$ data

Model fitted

- BR model: $p_{\rm d} = w_{\rm d}\rho_{\rm d}$

$$E(a)^2 = \Omega_{\rm r} a^{-4} + \Omega_{\rm m} a^{-3} + \Omega_{\rm d} a^{-3(1+w_{\rm d})}.$$

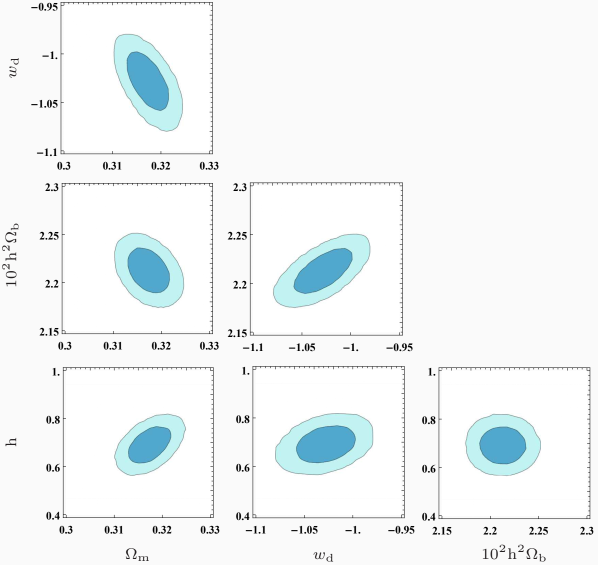
- LR model: $p_{\rm d} = -\left(\rho_{\rm d} + \beta\rho_{\rm d}^{\frac{1}{2}}\right)$

$$E^2(a) = \Omega_{\rm r} a^{-4} + \Omega_{\rm m} a^{-3} + \Omega_{\rm d}\left(1 + \frac{3}{2}\sqrt{\frac{\Omega_{\rm lr}}{\Omega_{\rm d}}}\ln(a)\right)^2.$$

- LSBR model: $p_{\rm d} = -\left(\rho_{\rm d} + \frac{\alpha}{3}\right)$

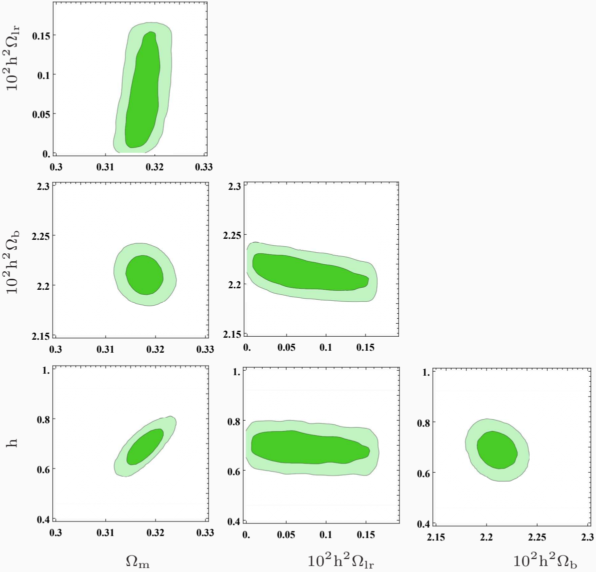
$$E^2(a) = \Omega_{\rm r} a^{-4} + \Omega_{\rm m} a^{-3} + \Omega_{\rm d}\left(1 - \frac{\Omega_{\rm lsbr}}{\Omega_{\rm d}}\ln(a)\right).$$

BR Model



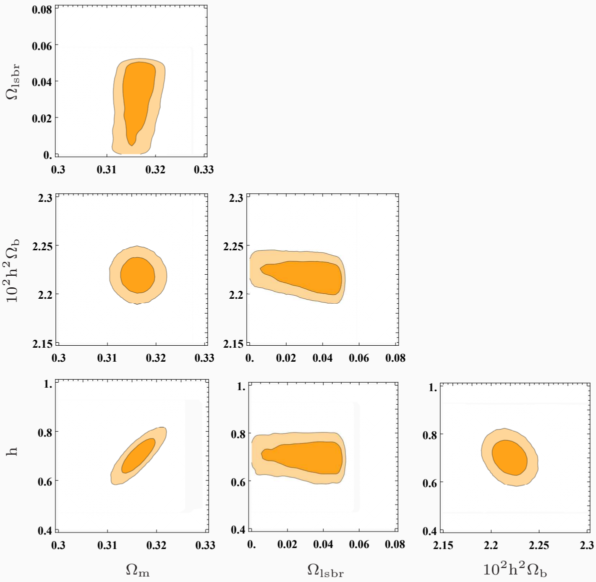
Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

LR Model



Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

LSBR Model



Bouali, Albarran, Bouhmadi-López and Ouali, arXiv:1905.07304 [astro-ph.CO]. Published in Physics of the Dark Universe.

Comparison with LCDM

Model	Par	Best fit	Mean	$\chi^2_{\rm tot}$	${\chi^2_{\rm tot}}^{\rm red}$	AIC_c	ΔAIC_c
ΛCDM	$\Omega_{\rm m}$	$0.318349^{+0.00248001}_{-0.00248001}$	$0.31834^{+0.00248987}_{-0.00248987}$	1047.42	0.957422	1053.441953	0
	h	$0.69814^{+0.0480814}_{-0.0480814}$	$0.698602^{+0.0481787}_{-0.0481787}$				
	$\Omega_{\rm b}h^2$	$0.022218^{+0.000120872}_{-0.000120872}$	$0.0222202^{+0.000122619}_{-0.000122619}$				
BR	$\Omega_{\rm m}$	$0.317173^{+0.00318473}_{-0.00318473}$	$0.317327^{+0.0031808}_{-0.0031808}$	1047.51	0.958380	1055.54663	2.104677
	$w_{\rm br}$	$-1.02758^{+0.0240102}_{-0.0240102}$	$-1.02874^{+0.0239306}_{-0.0239306}$				
	h	$0.691013^{+0.0507771}_{-0.0507771}$	$0.691523^{+0.0507536}_{-0.0507536}$				
	$\Omega_{\rm b}h^2$	$0.0221218^{+0.000170789}_{-0.000170789}$	$0.022123^{+0.000170538}_{-0.000170538}$				
LR	$\Omega_{\rm m}$	$0.317198^{+0.00276851}_{-0.00276851}$	$0.317705^{+0.00280131}_{-0.00280131}$	1047.53	0.958398	1055.56663	2.124677
	$\Omega_{\rm lr}$	$0.000445721^{+0.000416159}_{-0.000416159}$	$0.000763824^{+0.000416359}_{-0.000416359}$				
	h	$0.694604^{+0.0494111}_{-0.0494111}$	$0.688584^{+0.0493315}_{-0.0493315}$				
	$\Omega_{\rm b}h^2$	$0.0221295^{+0.000130585}_{-0.000130585}$	$0.0221028^{+0.000132755}_{-0.000132755}$				
LSBR	$\Omega_{\rm m}$	$0.317115^{+0.00253975}_{-0.00253975}$	$0.316144^{+0.00253899}_{-0.00253899}$	1047.56	0.958426	1055.59663	2.154677
	$\Omega_{\rm lsbr}$	$0.0500261^{+0.0130141}_{-0.0130141}$	$0.0299424^{+0.0133398}_{-0.0133398}$				
	h	$0.695705^{+0.0481201}_{-0.0481201}$	$0.701962^{+0.0481465}_{-0.0481465}$				
	$\Omega_{\rm b}h^2$	$0.022138^{+0.000121724}_{-0.000121724}$	$0.0221928^{+0.000121811}_{-0.000121811}$				

Table III. Summary of the best fit and the mean values of the cosmological parameters.

Late-time acceleration of the Universe within GR and with a phantom fluid

A perturbative approach: GR and phantom fluids

Our approach

- We start considering that the late-time acceleration of the universe is described by a dark energy component effectively encapsulated within a perfect fluid with energy density ρ_d and pressure p_d. On this setup, we consider two simple scenario:
  - A constant equation of state for DE
  - A DE in an effective and genuinely phantom DE universe. The reason of this second choice will become clear after considering the first case.
- Of course, on top of this we invoke a dark matter component.
- Given that to get the matter power spectrum, we start our numerical integration since the radiation dominated epoch, we will consider as radiation as well on our model.

Cosmological perturbations: GR and for the late Universe-1

- We worked on the Newtonian gauge and carried the first order perturbations considering DM, DE and radiation on GR. Radiation was included because our numerical integrations start from well inside the radiation dominated epoch (to get the matter power spectrum)
- We assumed initial adiabatic conditions for the different fractional energy density perturbations
- The total fractional energy density is fixed by Planck measurments; i.e. through A_s and n_s
- The speed of sound for DE:
  - The pressure perturbation of DE reads: $\delta p_d = c_{sd}^2 \delta\rho_d - 3\mathcal{H}(1+w_d)\left(c_{sd}^2 - c_{ad}^2\right)\rho_d v_d$, where $c_{sd}^2 = \left.\frac{\delta p_d}{\delta \rho_d}\right|_{r.f.}$ and $c_{aA}^2 = \frac{p_d'}{\rho_d'}$
  - Given that c_{sd}^2 is negative, we can end up with a problem (this is not intrisic to phantom matter as it can happen for example with fluids with a negative constant equation of state larger than -1)
  - We choose $c_{sd}^2 = 1$ as a phenomenological parameter

Cosmological perturbations: GR and for the late Universe-3

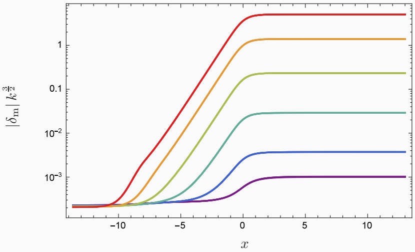
- Adiabatic conditions:

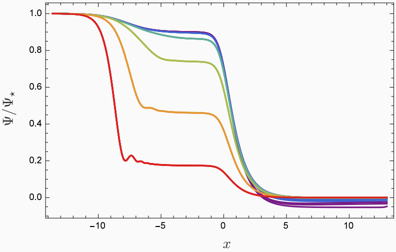
$$\frac{3}{4}\delta_{\rm r,ini} = \delta_{\rm m,ini} = \frac{\delta_{\rm d,ini}}{1 + w_{\rm d,ini}} \approx \frac{3}{4}\delta_{\rm ini}$$

$$v_{\rm r,ini} = v_{\rm m,ini} = v_{\rm d,ini} \approx \frac{\delta_{\rm ini}}{4\mathcal{H}_{\rm ini}}$$

- Initial conditions for δ are fixed through the amplitude and spectral index of the primordial inflationary power spectrum: $A_s = 2.143 \times 10^{-9}$, $n_s = 0.9681$ and $k_* = 0.05$ Mpc^{-1} (Planck values): $\Phi_{\rm ini} = \frac{2\pi}{3}\sqrt{2A_s}\left(\frac{k}{k_*}\right)^{n_s - 1} k^{-3/2}$
- Well inside the radiation era: $\Phi_{\rm ini} \approx -\frac{1}{2}\delta_{\rm tot,ini}$ and $\Phi_{\rm ini} \approx -2\mathcal{H}_{\rm ini} v_{\rm tot,ini}$
- We choose $c_{sd}^2 = 1$ as a phenomenological parameter
- The parameters of the models will be fixed through the fitting we did previously.

Results: DM perturbations and the gravitational potential





$k_1 = 3.33 \times 10^{-4}\mathrm{h\ Mpc}^{-1}$,

$k_2 = 1.04 \times 10^{-4}\mathrm{h\ Mpc}^{-1}$,

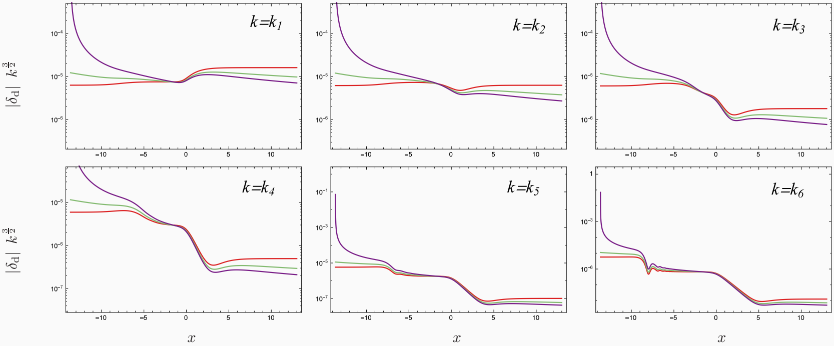
$k_3 = 3.26 \times 10^{-3}\mathrm{h\ Mpc}^{-1}$,

$k_4 = 1.02 \times 10^{-2}\mathrm{h\ Mpc}^{-1}$,

$k_5 = 3.19 \times 10^{-2}\mathrm{h\ Mpc}^{-1}$,

$k_6 = 1.00 \times 10^{-1}\mathrm{h\ Mpc}^{-1}$.

Results: DE perturbations

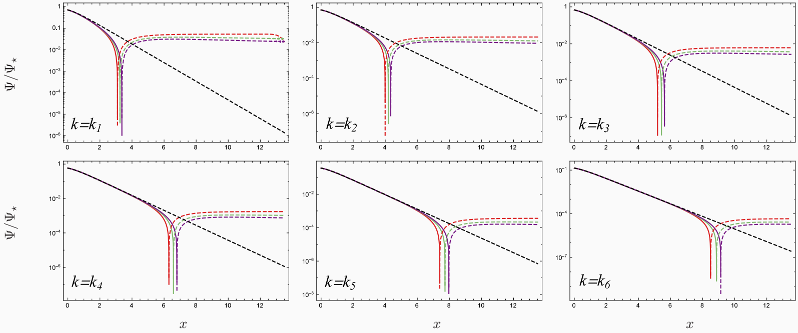


$k_1 = 3.33 \times 10^{-4} \mathrm{h\ Mpc}^{-1}$, $k_4 = 1.02 \times 10^{-2} \mathrm{h\ Mpc}^{-1}$,

$k_2 = 1.04 \times 10^{-4} \mathrm{h\ Mpc}^{-1}$, $k_5 = 3.19 \times 10^{-2} \mathrm{h\ Mpc}^{-1}$,

$k_3 = 3.26 \times 10^{-3} \mathrm{h\ Mpc}^{-1}$, $k_6 = 1.00 \times 10^{-1} \mathrm{h\ Mpc}^{-1}$.

Results: a closer look at the gravitational potential

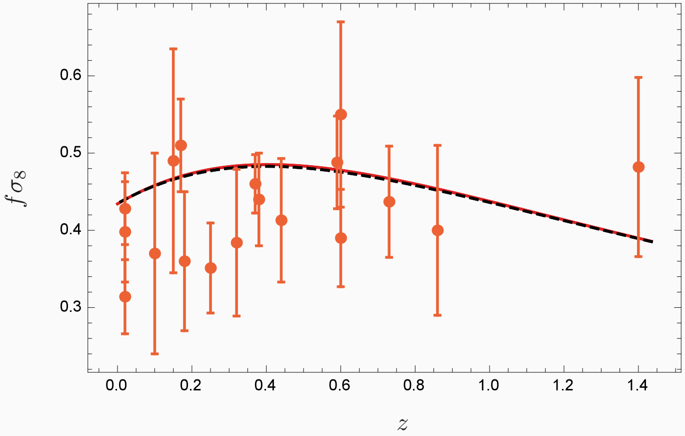


$$k_1 = 3.33 \times 10^{-4}\mathrm{h\ Mpc}^{-1}, \qquad k_4 = 1.02 \times 10^{-2}\mathrm{h\ Mpc}^{-1},$$

$$k_2 = 1.04 \times 10^{-4}\mathrm{h\ Mpc}^{-1}, \qquad k_5 = 3.19 \times 10^{-2}\mathrm{h\ Mpc}^{-1},$$

$$k_3 = 3.26 \times 10^{-3}\mathrm{h\ Mpc}^{-1}, \qquad k_6 = 1.00 \times 10^{-1}\mathrm{h\ Mpc}^{-1}.$$

Results: The evolution of $f\sigma_8$ (growth rate)-1-

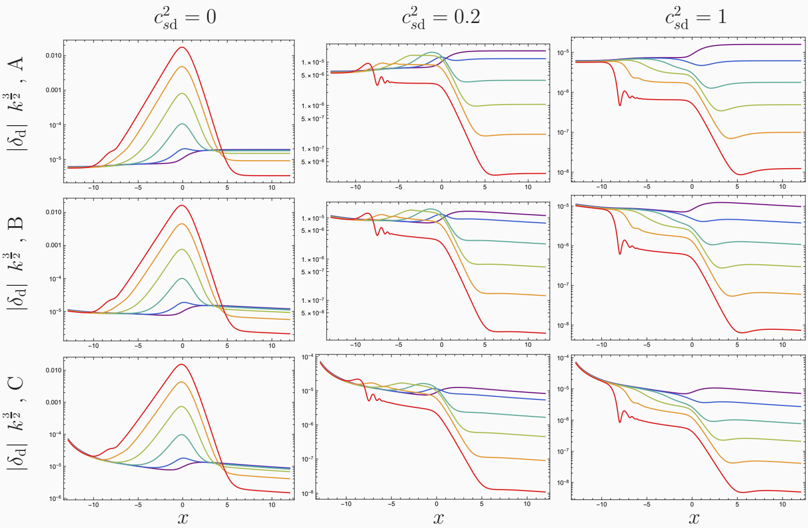


The evolution of $f\sigma_8$ for the 3 models.

$$f \equiv \frac{d\left(\ln \delta_{\mathrm{m}}\right)}{d\left(\ln a\right)}, \qquad \sigma_8\left(z,\ k_{\sigma_8}\right) = \sigma_8\left(0,\ k_{\sigma_8}\right) \frac{\delta_{\mathrm{m}}\left(z,\ k_{\sigma_8}\right)}{\delta_{\mathrm{m}}\left(0,\ k_{\sigma_8}\right)}$$

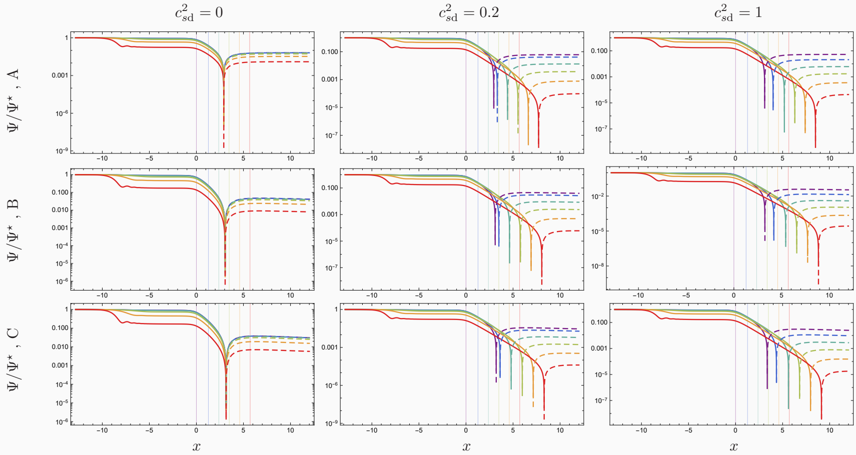
$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\ k_{\sigma_8}) = 0.820$ (Planck)

Effect of the speed of sound, C_{sd}^2, on DE perturbations

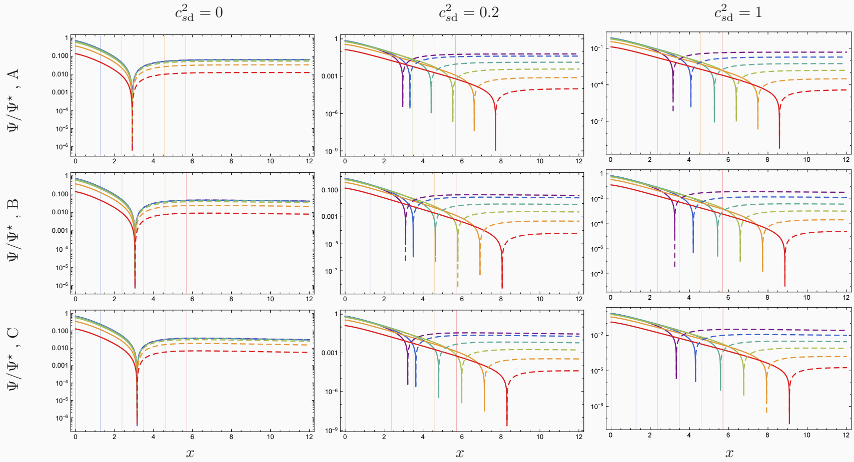


Albarran, Bouhmadi-López and Marto, arXiv:2011.08222 [gr-qc.CO]. Published in European Physical Journal C.

Effect of the speed of sound, C_{sd}^2, on the gravitational potential-1-



Effect of the speed of sound, C_{sd}^2, on the gravitational potential-2-



Late-time acceleration of the Universe within GR and with a phantom fluid

DE and DM models with interactions

DE and DM models with interaction

♯Motivations

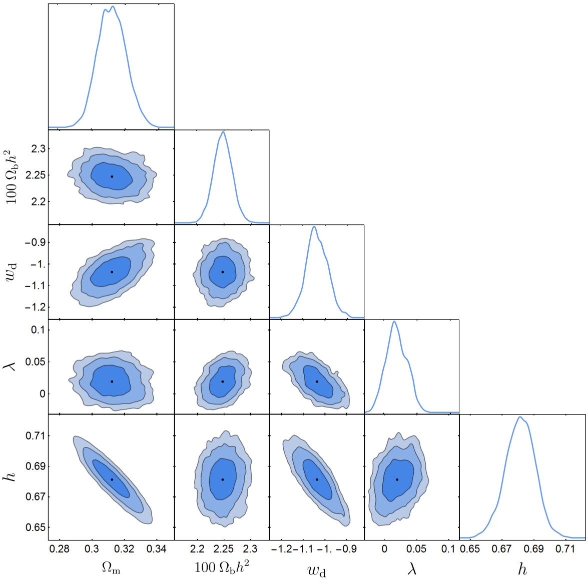
- Solving the coincidence problem.
- Check if an interaction between DE and DM could statistically improve the previous models.
- Check if the interaction could attenuate or modify the nature of future cosmological events induced by the former models corresponding to BR, LR and LSBR.
- Check whether the previous models A, B and C might respond differently to the interaction, as they exhibited very similar behaviour both at the background level and at the perturbative level.

$$\begin{cases} \dot{\rho}_{\rm m} + 3H\rho_{\rm m} = -Q, \\ \dot{\rho}_{\rm d} + 3H(1+w_{\rm d})\rho_{\rm d} = Q. \end{cases}$$

where

$$Q = \lambda H \rho_{\rm d}$$

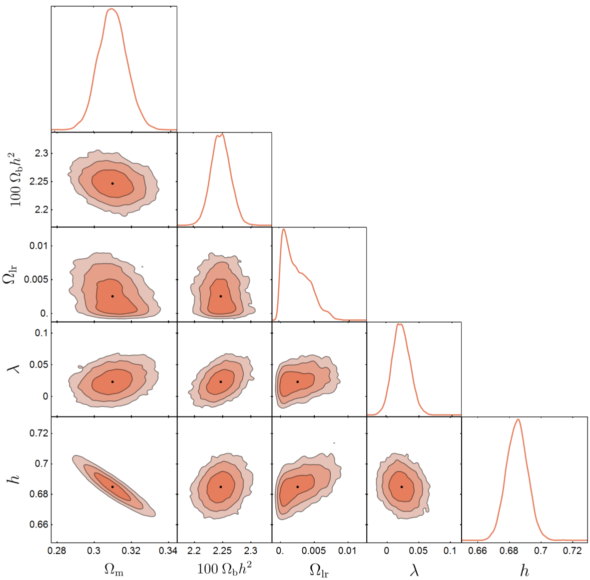
IBR Model-1-


2.3
2.25
2.2
$100\,\Omega_b h^2$
−0.9
−1.
−1.1
−1.2
w_d
0.1
0.05
0
λ
0.71
0.69
0.67
0.65
h
0.28 0.3 0.32 0.34
Ω_m
2.2 2.25 2.3
$100\,\Omega_b h^2$
−1.2 −1.1 −1. −0.9
w_d
0 0.05 0.1
λ
0.65 0.67 0.69 0.71
h

IBR Model-2-

Model	Par	Best fit	Mean
IBR	$\Omega_{\rm m}$	$0.310664^{+0.00830269}_{-0.00830269}$	$0.31219^{+0.00828119}_{-0.00828119}$
	$w_{\rm d}$	$-1.03248^{+0.0482072}_{-0.0482072}$	$-1.03728^{+0.0482912}_{-0.0482912}$
	λ	$0.0168888^{+0.0154053}_{-0.0154053}$	$0.0193522^{+0.0155017}_{-0.0155017}$
	h	$0.682033^{+0.0092436}_{-0.0092436}$	$0.681365^{+0.00922296}_{-0.00922296}$
	$\Omega_b h^2$	$0.0224867^{+0.000166489}_{-0.000166489}$	$0.0224764^{+0.000166426}_{-0.000166426}$

ILR Model-1-

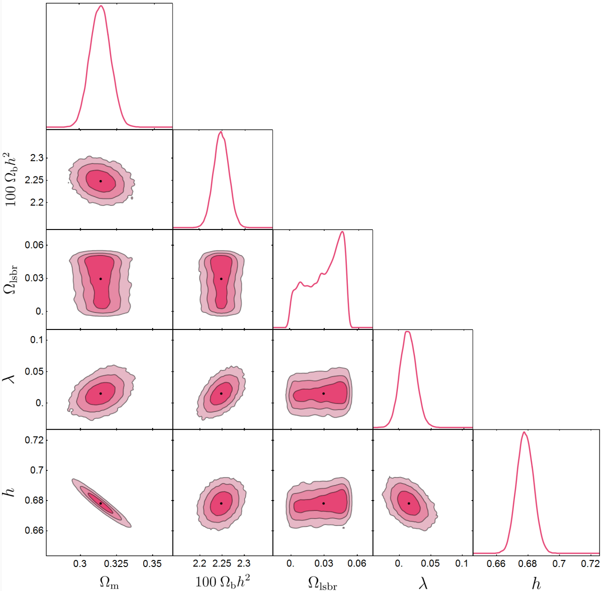


2.3
2.25
2.2
$100\,\Omega_b h^2$
0.01
0.005
0.
Ω_{lr}
0.1
0.05
0
λ
0.72
0.7
0.68
0.66
h
0.28
0.3
0.32
0.34
Ω_m
2.2
2.25
2.3
$100\,\Omega_b h^2$
0.
0.005
0.01
Ω_{lr}
0
0.05
0.1
λ
0.66
0.68
0.7
0.72
h

ILR Model-2-

Model	Par	Best fit	Mean
ILR	$\Omega_{\rm m}$	$0.310516^{+0.00726847}_{-0.00726847}$	$0.309924^{+0.00726091}_{-0.00726091}$
	$\Omega_{\rm lr}$	$0.00105782^{+0.00182933}_{-0.00182933}$	$0.00252318^{+0.0018261}_{-0.0018261}$
	λ	$0.017768^{+0.0129577}_{-0.0129577}$	$0.0231745^{+0.0129438}_{-0.0129438}$
	h	$0.682915^{+0.00661987}_{-0.00661987}$	$0.684813^{+0.00661268}_{-0.00661268}$
	$\Omega_b h^2$	$0.0224682^{+0.00016143}_{-0.00016143}$	$0.0224685^{+0.000161631}_{-0.000161631}$

ILSBR Model-1-



ILSBR Model-2-

Model	Par	Best fit	Mean
	$\Omega_{\rm m}$	$0.310516^{+0.00726847}_{-0.00726847}$	$0.309924^{+0.00726091}_{-0.00726091}$
	$\Omega_{\rm lr}$	$0.00105782^{+0.00182933}_{-0.00182933}$	$0.00252318^{+0.0018261}_{-0.0018261}$
ILR	λ	$0.017768^{+0.0129577}_{-0.0129577}$	$0.0231745^{+0.0129438}_{-0.0129438}$
	h	$0.682915^{+0.00661987}_{-0.00661987}$	$0.684813^{+0.00661268}_{-0.00661268}$
	$\Omega_b h^2$	$0.0224682^{+0.00016143}_{-0.00016143}$	$0.0224685^{+0.000161631}_{-0.000161631}$

Comparing Models-1-

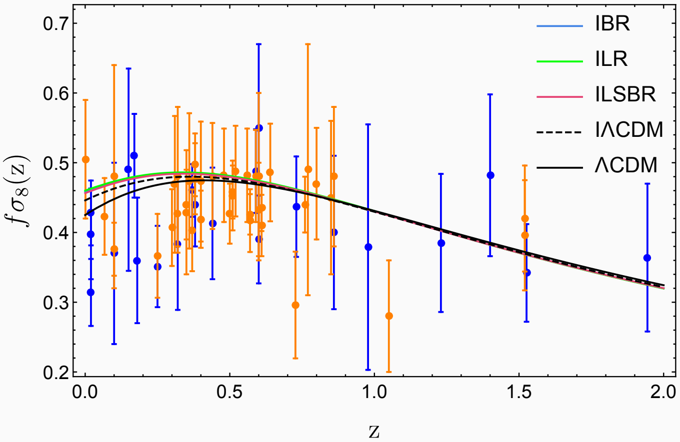
Model	Par	Best fit	Mean
ΛCDM	$\Omega_{\rm m}$	$0.312308^{+0.00607812}_{-0.00607812}$	$0.312583^{+0.00607362}_{-0.00607362}$
	h	$0.678603^{+0.00447778}_{-0.00447778}$	$0.678435^{+0.00447417}_{-0.00447417}$
	$\Omega_b h^2$	$0.0224102^{+0.000134672}_{-0.000134672}$	$0.0224002^{+0.00013449}_{-0.00013449}$
IΛCDM	$\Omega_{\rm m}$	$0.314738^{+0.00663245}_{-0.00663245}$	$0.315252^{+0.00663212}_{-0.00663212}$
	h	$0.676461^{+0.00504971}_{-0.00504971}$	$0.675976^{+0.005048282}_{-0.005048282}$
	λ	$0.00992076^{+0.0116578}_{-0.0116578}$	$0.011897^{+0.011643}_{-0.011643}$
	$\Omega_b h^2$	$0.0224698^{+0.000159949}_{-0.000159949}$	$0.0224845^{+0.000159746}_{-0.000159746}$
IBR	$\Omega_{\rm m}$	$0.310664^{+0.00830269}_{-0.00830269}$	$0.31219^{+0.00828119}_{-0.00828119}$
	$w_{\rm d}$	$-1.03248^{+0.0482072}_{-0.0482072}$	$-1.03728^{+0.0482912}_{-0.0482912}$
	λ	$0.0168888^{+0.0154053}_{-0.0154053}$	$0.0193522^{+0.0155017}_{-0.0155017}$
	h	$0.682033^{+0.0092436}_{-0.0092436}$	$0.681365^{+0.00922296}_{-0.00922296}$
	$\Omega_b h^2$	$0.0224867^{+0.000166489}_{-0.000166489}$	$0.0224764^{+0.000166426}_{-0.000166426}$
ILR	$\Omega_{\rm m}$	$0.310516^{+0.00726847}_{-0.00726847}$	$0.309924^{+0.00726091}_{-0.00726091}$
	$\Omega_{\rm lr}$	$0.00105782^{+0.00182933}_{-0.00182933}$	$0.00252318^{+0.0018261}_{-0.0018261}$
	λ	$0.017768^{+0.0129577}_{-0.0129577}$	$0.0231745^{+0.0129438}_{-0.0129438}$
	h	$0.682915^{+0.00661987}_{-0.00661987}$	$0.684813^{+0.00661268}_{-0.00661268}$
	$\Omega_b h^2$	$0.0224682^{+0.00016143}_{-0.00016143}$	$0.0224685^{+0.000161631}_{-0.000161631}$
ILSBR	$\Omega_{\rm m}$	$0.313552^{+0.00671554}_{-0.00671554}$	$0.314018^{+0.00670695}_{-0.00670695}$
	Ω_{lsbr}	$0.0458419^{+0.0144588}_{-0.0144588}$	$0.0295684^{+0.0145073}_{-0.0145073}$
	λ	$0.0154082^{+0.0120793}_{-0.0120793}$	$0.0148823^{+0.0120577}_{-0.0120577}$
	h	$0.679071^{+0.00524713}_{-0.00524713}$	$0.678149^{+0.00524195}_{-0.00524195}$
	$\Omega_b h^2$	$0.0224517^{+0.000162054}_{-0.000162054}$	$0.0224808^{+0.000161849}_{-0.000161849}$

Comparing Models-2-

Model	$\chi^{2}_{min}{}^{tot}$	AIC_c	ΔAIC_c
ΛCDM	1073.9795	1080.0014	0
IΛCDM	1073.1076	1081.1443	1.1429
IBR	1072.6870	1082.7420	2.7406
ILR	1072.6477	1082.7028	2.7014
ILSBR	1072.6200	1082.7600	2.6685

Notice that All the interacting models will induce a BR!!!

Results: The evolution of $f\sigma_8$ (growth rate)-1-

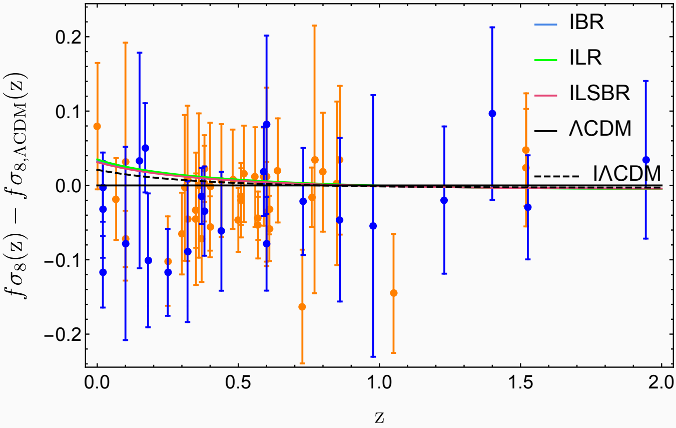


The evolution of $f\sigma_8$.

$$f \equiv \frac{d\,(\ln \delta_{\rm m})}{d\,(\ln a)}, \qquad \sigma_8\,(z,\ k_{\sigma_8}) = \sigma_8\,(0,\ k_{\sigma_8})\,\frac{\delta_{\rm m}\,(z,\ k_{\sigma_8})}{\delta_{\rm m}\,(0,\ k_{\sigma_8})}$$

$k_{\sigma_8} = 0.125$ hMpc^{-1}, $\sigma_8(0,\, k_{\sigma_8}) = 0.820$ (Planck)

Results: The evolution of $f\sigma_8$ (growth rate)-2-



Bouali, Albarran, Bouhmadi-López, Errahmani and Ouali, Phys. of The Dark Universe, arXiv:2103.13432[astro-ph.CO].

Late-time acceleration of the Universe within GR: A 3-form field

Can we have something more fundamental to describe DE?

- Can we have something more fundamental to describe phantom DE models?
    - A possibility come in the form of 3-forms.
    - Inspired in string theory: Copeland, Lahiri,Wands (1995)
    - Massless 3-form as Cosmological Constant (solving CC problem): Turok, Hawking (1998)
    - Inflation or late time acceleration driven by self-interacting 3-forms: Koivisto, Nunes (2009) and (2010)
    - Non-Gaussianity: Kumar, Mulryne, Nunes, Marto, Moniz (2016)
    - Quantum cosmology with 3-forms: Bouhmadi-López, Brizuela, Garay (2018)
- The answer as we will see in a momment is yes: Phantom DE models (LSBR): Morais, Bouhmadi-López, Kumar, Marto, Tavakoli (2017) and Bouhmadi-López, Marto, Morais and Silva (2017)

Late-time acceleration of the Universe within GR: A 3-form field

Reviewing the 3-form field $A_{\mu\nu\rho}$

p-forms

A p-form is a totally anti-symmetric covariant tensor:

$$\omega_{\mu_1 \ldots \mu_p} = \omega_{[\mu_! \ldots \mu_p]} \,.$$

In D-dimensions, the number of degrees of freedom of a massive p-form is

$$\text{degrees of freedom} = \frac{(D-1)!}{(D-1-p)!p!} \,.$$

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
Section based on Morais, B.L, Kumar, Marto and Tavakoli, Phys. Dark Univ. 15, 7 (2017) [arXiv:1608.01679 [gr-qc]]

p-forms in Cosmology

In a 4-dimensional space-time:

- $p = 0$ (scalar field) $\Rightarrow$ 1 degree of freedom
- $p = 1$ (vector field) $\Rightarrow$ 3 degrees of freedom
- $p = 2 \Rightarrow$ 3 degrees of freedom
- $p = 3 \Rightarrow$ 1 degree of freedom

$\Rightarrow$ The scalar field and the 3-form are the only ones compatible with a homogeneous and isotropic universe (in an easy way).

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)

The 3-form action

- We will consider the following action for a massive 3-form, $A_{\mu\nu\rho}$, minimally coupled to gravity

$$S^A = \int \mathrm{d}^4\mathbf{x}\sqrt{|\det g_{\mu\nu}|}\left[-\frac{1}{48}F^{\mu\nu\rho\sigma}F_{\mu\nu\rho\sigma} - V\left(A^{\mu\nu\rho}A_{\mu\nu\rho}\right)\right].$$

- The strength tensor, a 4-form, is defined through the exterior derivative: $F_{\mu\nu\rho\sigma} \equiv 4\nabla_{[\mu}A_{\nu\rho\sigma]}$
- The equation of motion, obtained from variation of S^A, is

$$\nabla_\sigma F^\sigma{}_{\mu\nu\rho} - 12\frac{\partial V}{\partial\left(A^2\right)}A_{\mu\nu\rho} = 0$$

- $\Rightarrow$ a massless 3-form is equivalent to a cosmological constanst

C. Germani and A. Kehagias, J. Cosmol. Astropart. Phys. 2009, 28 (2009)
T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
M. Duff and P. Van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980)

3-form Cosmology

We consider a homogeneous and isotropic universe described by the Friedmann-Lemaître-Robertson-Walker line element

$$ds^2 = -dt^2 + a^2(t)\gamma_{ij}dx^i dx^j \,.$$

t - cosmic time, $\dot{\{\}} = d\{\}/dt$
a - scale factor
x^i - comoving spatial coordinates (roman indices run from 1 to 3).

Only the purely spatial components of the 3-form are dynamical:

$$A_{0ij} = 0\,, \qquad A_{ijk} = a^3(t)\chi(t)\epsilon_{ijk}\,.$$

T. S. Koivisto, D. F. Mota, and C. Pitrou, J. High Energy Phys. 2009, 92 (2009)
Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

3-form Cosmology: background equations

⇒ Friedmann Equation

$$3H^2 = \kappa^2 \rho_\chi = \kappa^2 \left[\frac{1}{2} \left(\dot{\chi} + 3H\chi \right)^2 + V(\chi^2) \right] .$$

⇒ Raychaudhuri equation

$$\dot{H} = -\frac{\kappa^2}{2} \left(\rho_\chi + P_\chi \right) = -\frac{\kappa^2}{2} \chi \frac{\partial V}{\partial \chi} .$$

A 3-form can show phantom-like behavior if $\partial V / \partial \chi^2 < 0$.

⇒ Equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + 3\dot{H}\chi + \frac{\partial V}{\partial \chi} = 0 .$$

3-form Cosmology: evolution of χ-1-

Combining the Raychaudhuri equation and the equation of motion for χ:

$$\ddot{\chi} + 3H\dot{\chi} + \left(1 - \frac{\chi^2}{\chi_c^2}\right)\frac{\partial V}{\partial \chi} = 0\,.$$

The static solutions are:

- the critical points of the potential: $\dfrac{\partial V}{\partial \chi} = 0$,
- the limiting points: $\chi = \pm\chi_c$.

Once inside the interval $[-\chi_c,\, \chi_c]$, the field χ evolves towards a local minimum of V. However. . .

3-form Cosmology: evolution of χ-2-

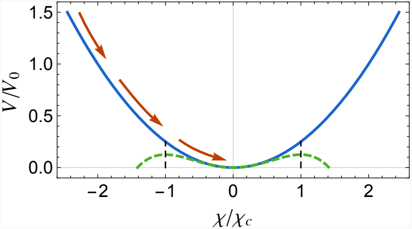
- Independently of the shape of a regular potential, in absence of DM interaction, the 3-form decays rapidly towards the interval $[-\chi_c, \chi_c]$ Koivisto and Nunes PLB [arXiv:0907.3883], idem PRD [arXiv:0908.0920]

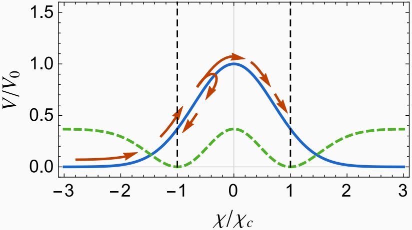
- In an expanding Universe, once inside the interval $[-\chi_c, \chi_c]$, the 3-form will end up in one of the minima of the potential (notice $V_{\rm eff} \neq V$).

- If the 3-form stops at the limits of this interval:

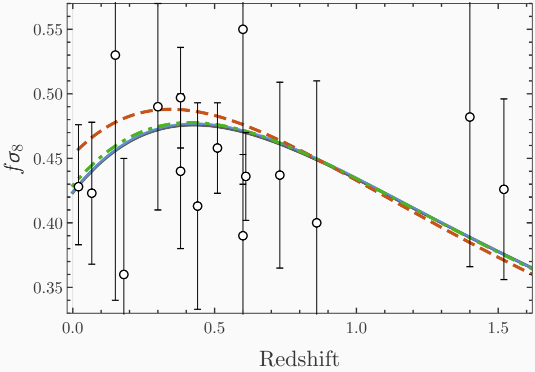
$$\chi = \pm\chi_c \quad \text{and} \quad \dot{\chi} = 0$$

- $\longrightarrow$ Universe heads towards a LSBR event $(\chi_c = \sqrt{2/3\kappa^2})$





Behaviour of fσ_8



Let me add that 3-forms can be quite interesting for further reasons as:

- They allow naturally for regular BHs (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2005.13260 [gr-qc]. Published in EPJC)
- They naturally support wormholes without changing the sign of the kinetic energy (Bouhmadi-López, Chen, Chew, Ong and Yeom, arXiv: 2108.07302 [gr-qc]. Published in JCAP

DE singularities in GR: a quantum approach

On the quantum fate of singularities in a dark-energy dominated universe

- There is no successful quantum gravity theory so far that would lead to THE theory of quantum cosmology
- There are, however, several approaches in this direction. Here we will follow the most conservative one which corresponds to the Wheeler deWitt approach.
- The Wheeler DeWitt equation is the equivalent to Schrödinger like equation

On the quantum fate of singularities in a dark-energy dominated universe within GR

Within the framework of quantum geometrodynamics and mainly within a Born Oppenheimer approximation

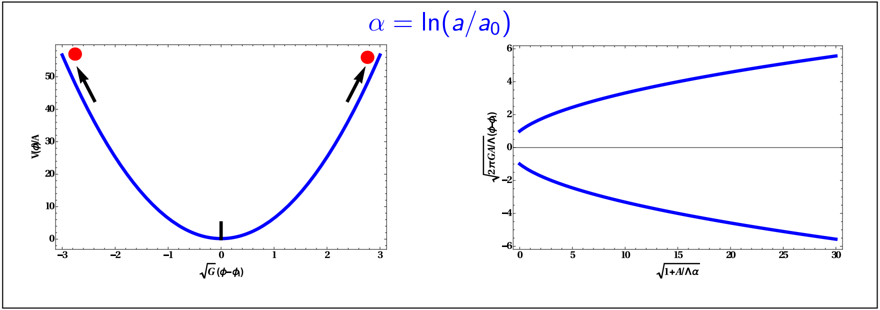
- It was shown that the big rip can be removed Dabrowski, Kiefer and Sandhöfer, PRD, [arXiv:hep-th/0605229], Alonso, B.L. and Martín-Moruno, PRD, [arXiv:1802.03290 [gr-qc]].
- It was shown the avoidance of a big brake singularity Kamenshchik, Kiefer and Sandhöfer 07', PRD, [arXiv:0705.1688].
- It was shown also the avoidance of a big démarrage singularity and a big freeze BL, Kiefer, Sandhöfer and Moniz, PRD, [arXiv:0905.2421]
- Type IV singularity is removed BL, Krämer and Kiefer, PRD, [arXiv:1312.5976].
- It has been shown as well that LR can be removed Albarran, BL, Kiefer, Marto, Moniz, PRD, [arXiv:1604.08365].

Review the on the topic by B.L., Kiefer and Martín-Moruno arXiv:1904.01836 (GRG)

LSBR driven by a scalar field

- Phantom scalar field ϕ: $\rho_\phi = -\frac{1}{2}\dot{\phi}^2 + V(\phi)$, $\quad p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi)$

$$\ddot{\phi} + 3H\dot{\phi} - V'(\phi) = 0, \quad V(\phi) = \frac{A}{6} + 2\pi AG\,(\phi - \phi_1)^2$$



LSBR: Quantisation with a scalar field

- The Wheeler-DeWitt equation:

$$\frac{\hbar^2}{2}\left[\frac{\kappa^2}{6}\frac{\partial^2}{\partial\alpha^2}+\frac{\partial^2}{\partial\phi^2}\right]\psi(\alpha,\phi)+a_0^6 e^{6\alpha}V(\phi)\psi(\alpha,\phi)=0$$

- Can be solved within the BO:
  - The gravitational part are oscillatory or exponential functions.
  - The matter part can be written as parabolic cylinder functions that decay to zero at large value of the scale factor.
- It can be shown that there are solutions (wave functions) that vanishe close to the classically abrupt event. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an "abrupt event" avoidance.

Albarran, BL, Cabral, Martín-Moruno, JCAP, [arXiv:1509.07398] (minimally coupled scalar field)
B.L., Brizuela and Garay, JCAP, [arXiv:1802.05164 [gr-qc]] (3-forms)

Borislavov Vasilev, B.L., Martín-Moruno, PRD, [arXiv:1907.13081 [gr-qc]] ($f(R)$ metric theories)

LSBR: Quantisation with a 3-form-1-

- The classical action for a FLRW universe (spatially flat) reads:

$$S = S_A + S_{EH} = \int dt\, \mathcal{V} N \left(\frac{-a\dot{a}^2}{2\kappa N^2} + \frac{\dot{\phi}^2}{2a^3 N^2} - a^3 V \right),$$

where $\phi = a^3 \chi$.

- The classical Hamiltonian for a FLRW universe (spatially flat) reads:

$$\mathcal{H} = N \left(-\frac{\kappa}{2a} p_a^2 + \frac{a^3}{2} p_\phi^2 + a^3 V \right),$$

where

$$p_a = \frac{\delta L}{\delta \dot{a}} = -\frac{a\dot{a}}{\kappa N}, \quad p_\phi = \frac{\delta L}{\delta \dot{\phi}} = \frac{\dot{\phi}}{a^3 N}.$$

B.L., Brizuela and Garay, JCAP, [arXiv:1802.05164 [gr-qc]] (3-forms)

LSBR: Quantisation with a 3-form-2-

- The classical Hamiltonian for a FLRW universe (spatially flat) can be rewritten as:

$$\mathcal{H} = N\left(\frac{1}{2}G^{AB}p_A p_B + a^3 V\left(6(a^{-3}\phi)^2\right)\right),$$

with A and B indices referring to a or ϕ and the mini-superspace metric given by

$$G^{AB} = \begin{pmatrix} -\frac{\kappa}{a} & 0 \\ 0 & a^3 \end{pmatrix}. \qquad G_{AB} = \begin{pmatrix} -\frac{a}{\kappa} & 0 \\ 0 & \frac{1}{a^3} \end{pmatrix}.$$

LSBR: Quantisation with a 3-form-3-

- With the usual prescription, we transform the classical dynamical variables into quantum operators by means of the Laplace-Beltrami operator:

$$G^{AB} p_A p_B \to -\frac{\hbar^2}{\sqrt{-G}} \partial_A (\sqrt{-G}\, G^{AB} \partial_B)\,,$$

where G is the determinant of G_{AB}.

- The Wheeler-DeWitt equation then reads:

$$\left(\hbar^2 \kappa \partial_\beta^2 - \hbar^2 \partial_\phi^2 + 2V\right) \psi(\beta, \phi) = 0,$$

and $\beta = a^3/3$.

LSBR: Quantisation with a 3-form-4-

- The potential:

$$V = V_0 e^{-\lambda^2\chi^2} = V_0 e^{-9\lambda^2\phi^2/\beta^2}\,,$$

  with $\lambda^2 = \kappa/2\sigma^2$, and σ the dimensionless width of the Gaussian potential.

- The WDW equation is given now by:

$$\left(\hbar^2\kappa\partial_\beta^2 - \hbar^2\partial_\phi^2 + 2V_0 e^{-9\lambda^2\phi^2/\beta^2}\right)\psi(\beta,\phi) = 0.$$

LSBR: Quantisation with a 3-form-5-

- The WdW eq. can be solved as follows:
  - The matter part can be written in different ways depending on the approximations used
    - Constant potential (exponential decreasing functions)
    - Linear approximation for the potential + a B.O. approximation (Airy functions)
    - quadratic approximation for the potential + a B.O. approximation (Bessel functions)
    - Full potential + a B.O. approximation and a WKB approximation (not on the paper, I notice when preparing this seminar)
  - The matter part decays to zero at large value of the scale factor.
  - The gravitational part are oscillatory or exponential (decaying) functions.
- It can be shown that there are solutions (wave functions) that vanishe close to the classically abrupt event. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an "abrupt event" avoidance.

B.L., Brizuela and Garay, JCAP, [arXiv:1802.05164 [gr-qc]] (3-forms)

DE singularities within modified theories of gravity (metric and Palatini examples): a quantum approach

DE singularities within modified theories of gravity (metric and Palatini examples): a quantum approach

A Metric road

$f(R)$-gravity: The action

The Einstein-Hilbert action is replaced by

$$S^{(f)} = \frac{1}{2\kappa^2} \int \mathrm{d}^4\mathbf{x} \sqrt{-g} f(R) , \qquad \kappa^2 = 8\pi G$$

where $f(R)$ is a generic function of the Ricci scalar R.

Among the general class of Modified Theories of Gravity, $f(R)$-gravity has been one of the most studied cases

- The simple equations of motion are easy to study;
- $f(R)$-gravity already captures interesting effects of MTG (e.g. the Starobinsky model for R^2 inflation);
- Possibility to avoid the solar system constraints (e.g. through the chameleon mechanism).

Nojiri and Odintsov, Int. J. Geom. Meth. Mod. Phys. 4, 115 (2007) [hep-th/0601213], Capozziello and De Laurentis, Phys. Rept. 509, 167 (2011) [arXiv:1108.6266 [gr-qc]].

$f(R)$-gravity: The different formalisms

Within the $f(R)$-theories of gravity we can identify different classes:

- metric formalism: the metric includes all the dynamical degrees of freedom.
- Palatini formalism: the connection Γ is independent of the metric g.
- metric-affine formalism: like the Palatini formalism but the matter Lagrangian density depends on both the metric and the connection.
- hybrid metric-Palatini formalism: a mixed action with terms that depend on the metric and the Levi-Civita connection and terms that depend on the metric and the independent connection.

$f(R)$-gravity: The modified Einstein equations

In the metric formalism, variation of the gravitational action with regards to the metric $g^{\mu\nu}$ leads to the modified Einstein equations:

$$R_{\mu\nu} f_R - \frac{1}{2} g_{\mu\nu} f - \left(\nabla_\mu \nabla_\nu - g_{\mu\nu} \Box\right) f_R = \kappa^2 T^{(m)}{}_{\mu\nu} \,,$$

$$T^{(m)}{}_{\mu\nu} := \frac{-2}{\sqrt{-g}} \frac{\delta \mathcal{L}^m}{\delta g^{\mu\nu}} \qquad \Box := g_{\mu\nu} \nabla_\mu \nabla_\nu \qquad f_R := \frac{df}{dR} \,.$$

The absence of extra coupling with the matter sector means that the usual matter conservation is verified

$$\nabla_\mu T^{(m)}{}^{\mu}_{\ \nu} = 0$$

Consider the FLRW metric

$$ds^2 = -dt^2 + \frac{\delta_{ij} dx^i dx^j}{1 - \frac{\mathcal{K}}{4}\left(x^2 + y^2 + z^2\right)}, \qquad \mathcal{K} = -1, 0, +1\,.$$

Taking the (00) component of Einstein equation, we get the Friedmann equation

$$3\left(H^2 + \frac{\mathcal{K}}{a^2}\right) f_R + \frac{1}{2}\left(f - f_R R\right) + 3H\dot{R} f_{RR} = \kappa^2 \rho_m\,,$$

complemented by the conservation equation

$$\dot{\rho}_m + 3H\left(\rho_m + P_m\right) = 0\,,$$

where $\dot{x} := (\partial x)/(\partial t)$.

A quantum approach within metric $f(R)$ theory

- The modified WdW equation (first obtained by Vilenkin back in the 80')

$$\left[\partial_q^2 - \frac{1}{q^2}\partial_x^2 - V(q,x)\right]\Psi(q,x) = 0,$$

where the potential is given by

$$V(q,x) = \frac{q^2}{\lambda^2}\left[k + \frac{f_{R0}}{6R_0}\left(f - Rf_R\right)\frac{q^2}{f_R^2}\right],$$

and

$$q = \sqrt{R_0}\, a \left(f_R/f_{R0}\right)^{1/2} \text{ and } x = \ln\left(f_R/f_{R0}\right)^{1/2},$$

- It is a PDE because in metric $f(R)$ theories there is an extra degree of freedom, the scaleron.

$f(R)$ quantum cosmology and the BR

- A suitable choice would be

$$f(R) = \alpha_+ R^\gamma, \qquad \text{with} \qquad \gamma = 2 + \sqrt{3/2},$$

- Then

$$V(q,x) = -\frac{A}{\lambda^2} e^{-Bx} q^4,$$

$$q = \sqrt{R_0}\, a \left(R/R_0\right)^{\frac{\gamma-1}{2}}, \quad x = \ln\left(R/R_0\right)^{\frac{\gamma-1}{2}},$$

where $A = \frac{\gamma-1}{6\gamma} = \frac{1}{30}(1+\sqrt{6})$, and $B = 2\frac{\gamma-2}{\gamma-1} = 6 - 2\sqrt{6}$.

- The WdW equation becomes

$$\left[q^2 \partial_q^2 - \partial_x^2 + \frac{A}{\lambda^2} e^{-Bx} q^6\right] \Psi(q,x) = 0.$$

- It cannot be solved exactly but it can be shown there are approximate solutions (wave functions) that vanishe close to the classically singularity. Therefore, the DeWitt condition is fullfilled. This result can be interpreted as an singularity avoidance.

Alonso-Serrano, Bouhmadi-López and Martín-Moruno, PRD [arXiv:1802.03290 [gr-qc]].

DE singularities within modified theories of gravity (metric and Palatini examples): a quantum approach

A Palatini approach

DE singularities within modified theories of gravity: a quantum approach within EiBI theory-1-

- There have been many proposals for alternative theories of GR as old as the theory itself
- One of the oldest proposal was due to Eddington
- In Eddington proposal, the connection rather than the metric plays the fundamental role of the theory
- It is equivalent to GR in vacuum
- BUT does not incorporate matter
- An Eddington-inspired-Born-Infeld theory has been proposed by Bañados and Ferreira

Bañados and Ferreira, PRL, [arXiv:1006.1769]

EiBI theory-2-

$$\mathcal{S}_{\rm EiBI}(g,\Gamma,\Psi) = \frac{2}{\kappa}\int d^4x \left[\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{|g|}\right] + \mathcal{S}_{\rm m}(g,\Gamma,\Psi)$$

- We consider the action under the Palatini formalism, i.e., the connection $\Gamma^{\alpha}_{\mu\nu}$ is *not* the Levi-Civita connection of the metric $g_{\mu\nu}$
- This Lagrangian has two well defined limits: (i) when $|\kappa R|$ is very large, we recover Eddington's theory and (ii) when $|\kappa R|$ is small, we obtain the Hilbert-Einstein action with an effective cosmological constant $\Lambda = (\lambda - 1)/\kappa$
- A solution of the above action can be characterized by two different Ricci tensors: $R_{\mu\nu}(\Gamma)$ as presented on the action and $R_{\mu\nu}(g)$ constructed from the metric g
- There are in addition three ways of defining the scalar curvature. These are: $g^{\mu\nu}R_{\mu\nu}(g)$, $g^{\mu\nu}R_{\mu\nu}(\Gamma)$ and $R(\Gamma)$. The third one is derived from the contraction between $R_{\mu\nu}(\Gamma)$ and the metric compatible with the connection Γ
- Therefore, whenever, we refer to singularity avoidance, one must specify the specific scalar curvature(s). This will affect the way we apply the DeWitt criterium

EiBI theory-3-

- Gravitational action:

$$\mathcal{S}_{\rm EiBI}(g,\Gamma,\Psi)=\frac{2}{\kappa}\int d^4x\left[\sqrt{|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)|}-\lambda\sqrt{|g|}\right]+\mathcal{S}_{\rm m}(g,\Gamma,\Psi)$$

- The parameter κ has been constrained using observationally for example from BBN (Casanellas *at al*, ApJ, arXiv:1109.0249, Avelino, PRD, arXiv:1201.2544).
- The model can avoid the Big Bang singularity, for example, in a radiation dominated universe (Bañados and Ferreira, PRL, arXiv:1006.1769 + 2012).
- Has been proposed as an alternative scenario to the inflationary paradigm (Avelino, PRD, arXiv:1205.6676)
- It was shown that if the null energy condition is fullfilled then the *apparent* null energy condition is also fullfilled (Deslate and Steinhoff, PRL, arXiv:1201.4989).
- Can this theory avoid the big rip singularity? Answer No (BL, Che-Yu Chen, Pisin Chen, EPJC, arXiv:1302.6249, EPJC, arXiv:1406.6157, PRD, arXiv:1407.5114, EPJC, arXiv:1507.00028)

Towards EiBI quantisation-1-

- The equation of motion of EiBI theory can be equally obtain from the action (Delsate and Steinhoff, PRL, arXiv:1201.4989)

$$S_a = \lambda \int d^4x \sqrt{-q}\Big[R(\Gamma) - \frac{2\lambda}{\kappa} + \frac{1}{\kappa}(q^{\alpha\beta}g_{\alpha\beta} - 2\tau)\Big] + S_M(g),$$

- Unlike the EiBI action, this action is linear on $R(\Gamma)$. This makes the quantization much "simpler"
- The starting point is the homogeneous and isotropic ansatz of the Universe:

$$\begin{aligned} g_{\mu\nu}dx^\mu dx^\nu &= -N(t)^2dt^2 + a(t)^2 d\vec{x}^2, \\ q_{\mu\nu}dx^\mu dx^\nu &= -M(t)^2dt^2 + b(t)^2 d\vec{x}^2. \end{aligned}$$

where $N(t)$ and $M(t)$ are the lapse functions of $g_{\mu\nu}$ and $q_{\mu\nu}$, respectively.

Towards EiBI quantisation-2-

- The Lagrangian reads

$$\mathcal{L} = \lambda M b^3 \Big[-\frac{6\dot{b}^2}{M^2 b^2} - \frac{2\lambda}{\kappa} + \frac{1}{\kappa}\Big(\frac{N^2}{M^2} + 3\frac{a^2}{b^2} - 2\frac{Na^3}{Mb^3}\Big)\Big] - 2\rho(a) N a^3,$$

- The Hamiltonian reads (as there is no singularity at $b = 0$ for the model, we can safely rescale the Hamiltonian as)

$$b^3 \mathcal{H} = M\Big[-\frac{b^2 p_b^2}{24\lambda} + \frac{2\lambda^2}{\kappa} b^6 + \frac{1}{\kappa\lambda}(\lambda + \kappa\rho(a))^2 a^6 - \frac{3\lambda}{\kappa} a^2 b^4\Big] = 0$$

- Then, we can write down the WDW equation by choosing the following factor ordering:

$$b^2 p_b^2 = -\hbar^2 \Big(b\frac{\partial}{\partial b}\Big)\Big(b\frac{\partial}{\partial b}\Big) = -\hbar^2\Big(\frac{\partial}{\partial x}\Big)\Big(\frac{\partial}{\partial x}\Big)$$

where $x = \ln(\sqrt{\lambda} b)$.

Towards EiBI quantisation-3-

- Therefore, the WDW equation reads:

$$\left[\frac{\partial^2}{\partial x^2} + V_1(a,x)\right]\Psi(a,x) = 0,$$

where

$$V_1(a,x) = \frac{24}{\kappa\hbar^2}\left[2\mathrm{e}^{6x} - 3a^2\mathrm{e}^{4x} + (\lambda + \kappa\rho(a))^2 a^6\right].$$

- It can be shown that the wave function decays and vanishes when approaching $a \to \infty$.
- The DeWitt criterium is therefore fullfilled!!!

BL, Che-Yu Chen, JCAP, arXiv:1609.00700. See also our work, EPJC, arXiv:1911.03935

Conclusions

Conclusions

- We have followed a phenomenological approach to describe the late-time acceleration of the universe.
- We have also shown that the late-time acceleration of the Universe can be described through a phantom DE component
- We have looked for the observational fit and the perturbations
- We have described phantom DE through a more fundamental field encoded in a 3-form
- Then finally, we have shown using the WDW equation that the DE singularities or abrupt event can be unharmful in a quantum context.

Thank you for your attention !!!