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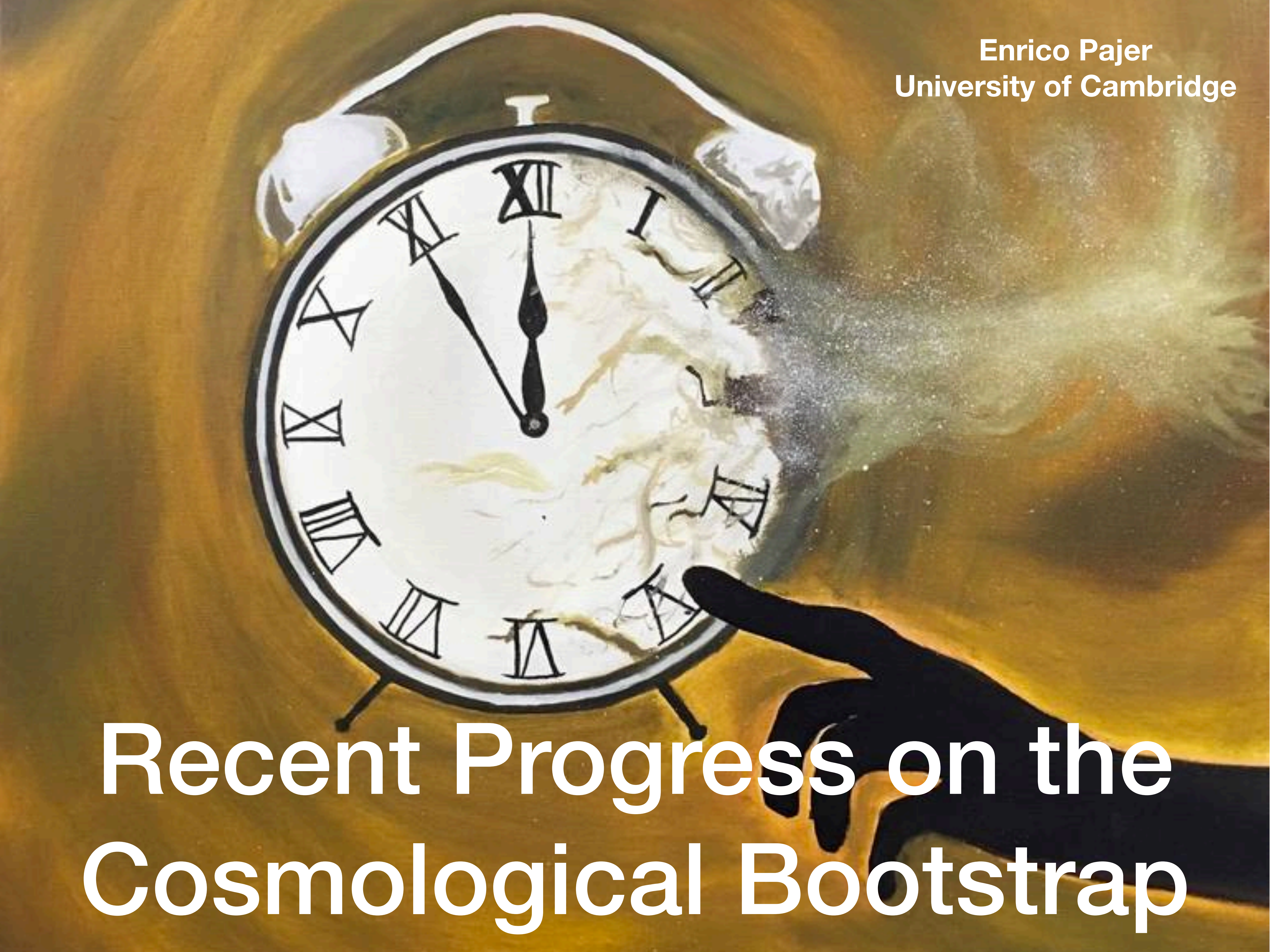
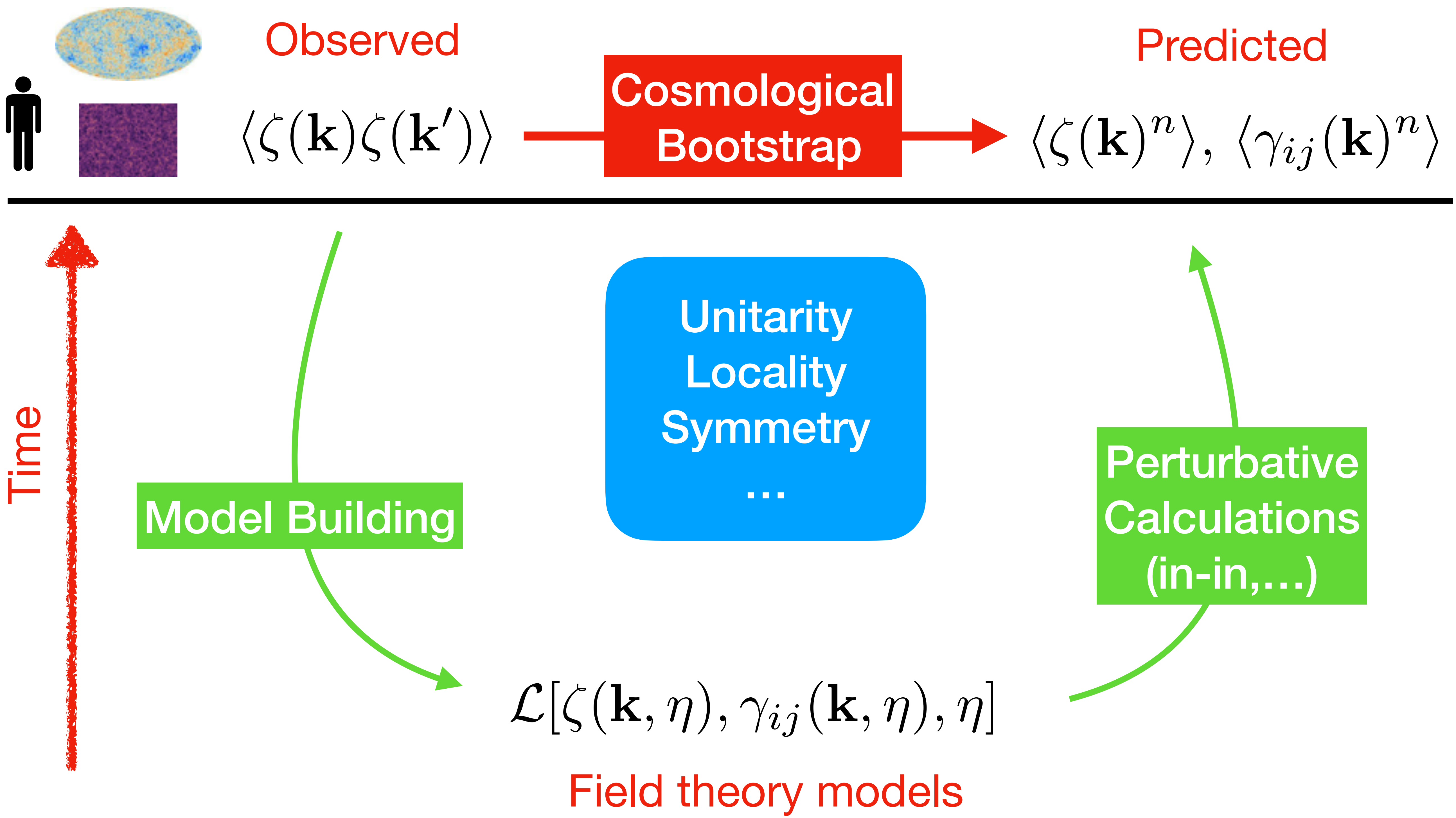


Recent Progress on the Cosmological Bootstrap

The Cosmological Bootstrap



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Primordial Cosmo = QFT in curved spacetime / Quantum Gravity

- On large scales (>> Mpc) cosmological surveys measure QFT correlators of metric fluctuation

$$\langle \prod^n \delta(k_a) \rangle \sim \int_k \left[\prod^n \Delta^{(Y)}(k_a) \right] \langle \prod^n \zeta(k_a) \rangle$$

CMB temperature
density of galaxies
dark matter, …

QFT / QG
in de Sitter

- The goal of primordial cosmology is to understand QFT and QG in (approximately, asymptotically) de Sitter

Aspirations

- We hope to learn about:
  - *New degrees of freedom and their interactions*: Inflation requires at least one degree of freedom and three energy scales beyond the standard model.
  - The laws of gravity at short distances/high energies: probe GR and beyond at high energies
  - QFT in de Sitter: which theories are consistent?
  - Quantum gravity in dS? Holography, string theory?

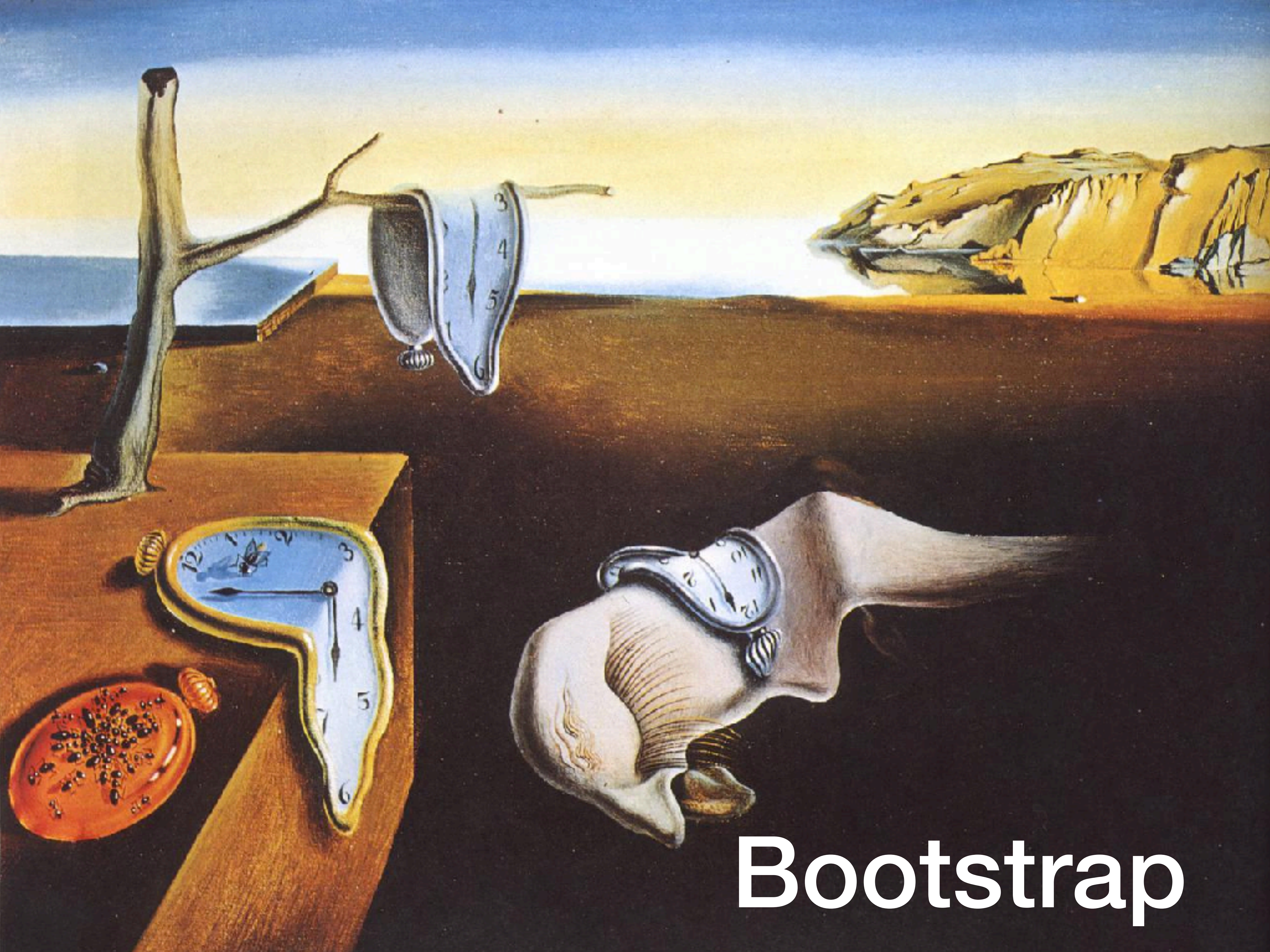
Problems

Here's some obstacles we are facing (in order of difficulty)

- *Too many models* for too little data on primordial universe. Concrete constructions model time evolution, which is not observable.
- Too little *theoretical guidance*: what are necessary conditions on EFT's to admit standard (e.g. unitary, causal,…) UV-completions?
- What's our estimate for non-perturbative effects from QFT and quantum gravity? Any guidance from bottom-up de Sitter holography?

To make progress, I will describe a new approach to compute observables in de Sitter and inflation, the *boostless cosmological bootstrap*. This gives us very general consequences of locality and unitarity within perturbative QFT on a curved background.

A poster-child of the approach is the calculation of all graviton non-Gaussianities



Bootstrap

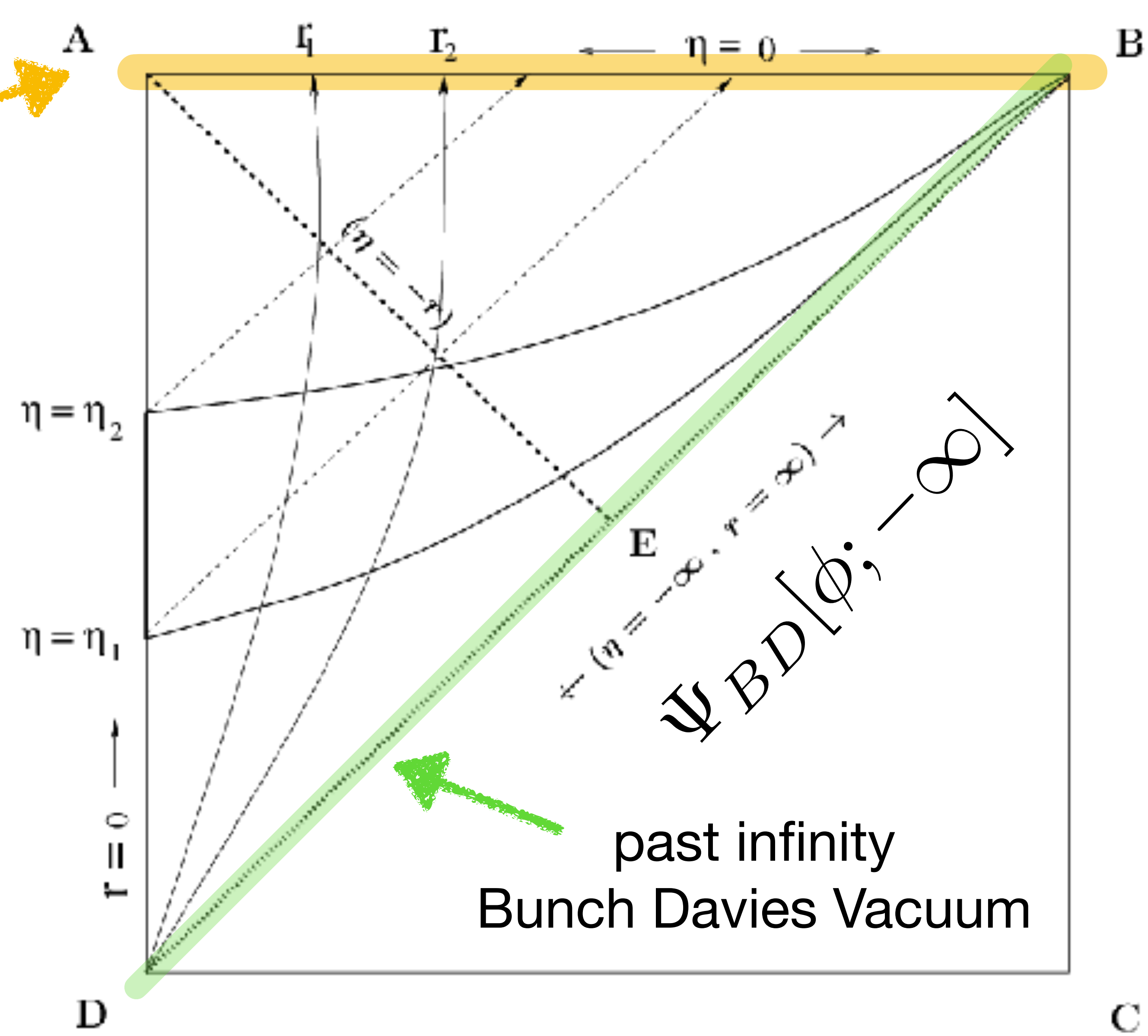
Penrose diagram

- We work in the Poincare' patch (half of dS)

$$ds^2 = -dt^2 + a^2 dx^2 = a^2(-d\eta^2 + dx^2)$$

$$\Psi[\phi; \eta \to 0]$$

The future (conformal) boundary can be thought as the reheating surface after inflation and determines the statistics of *LSS and CMB observations*



The wavefunction

- The *wavefunction of the universe,* is a functional of the all fields in the theory (including the metric) at some time:

$$\Psi[\phi,\eta] = \exp\left[\sum_{n=2}^{\infty}\int_{\mathbf{k}} \psi_n(\mathbf{k}_1,\ldots,\mathbf{k}_n;\eta)\prod_a^n \phi(\mathbf{k}_a)\right]$$

- All probabilities can be computed as in QM

$$\langle\mathcal{O}\rangle = \int d\phi\Psi^*\mathcal{O}\Psi$$

- The Ψ_n are closely related to *cosmological correlators*, which determine the statistics of the Cosmic Microwave Background and Large Scale Structures

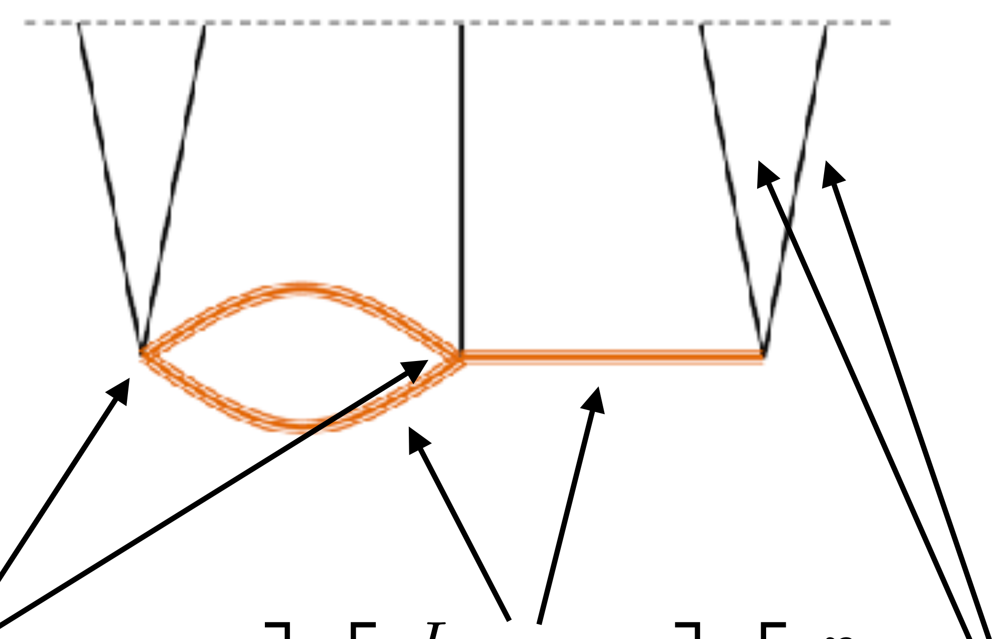
$$\langle\phi\phi\rangle = [2\mathrm{Re}\psi_2(k)]^{-1}\,, \qquad \langle\phi\phi\phi\rangle = -2\mathrm{Re}\psi_3\left[\prod^3 \mathrm{Re}\psi_2\right]^{-1}$$

Feynman Diagrams

- Given a model, the wavefunction can be computed perturbatively from a path integral

$$\Psi[\phi(\mathbf{x});\eta_0] = \int_{\text{vacuum}}^{\phi(\mathbf{x},\eta_0)} [\mathcal{D}\phi]\, e^{iS[\phi]}$$

with the following Feynman rules (equivalent to in-in):



$$\psi_n = \left[\prod_A^V \int d\eta_A\, F_A\right] \left[\prod_m^I G(p)\right] \left[\prod_a^n K(k_a)\right]$$

Propagators

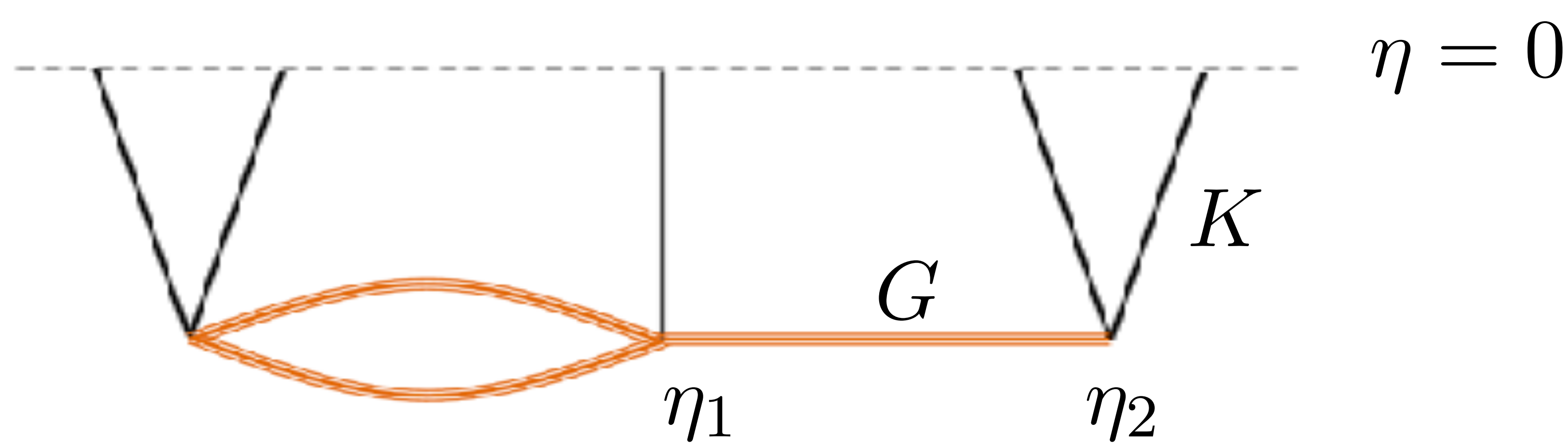
- Simples examples are the massless spin-2 (graviton) and spin-0 (scalar) fields:

$$K(\eta, k) = (1 - ik\eta)e^{ik\eta}\,, \qquad P(k) = \frac{H^2}{2k^3}$$

$$K(t, k) = e^{i\Omega t} = e^{i\sqrt{k^2+m^2}t} \qquad P(k) = \frac{1}{k}$$

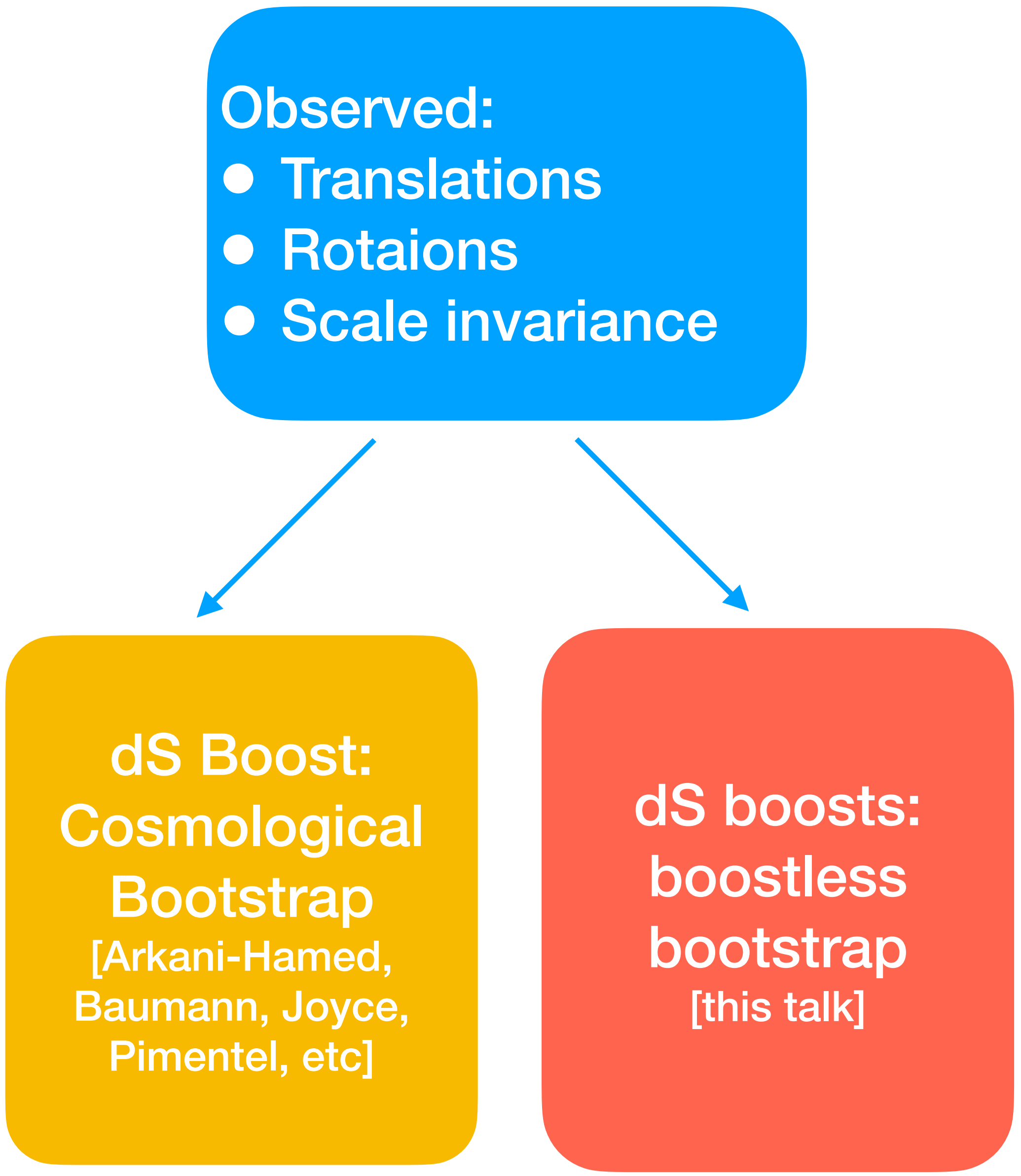
- Bulk-bulk propagator is Feynman propagator + boundary

$$G(\eta_1, \eta_2, k) = 2P(k)\left[\theta(\eta_1 - \eta_2)K(\eta_2, k)\,\mathrm{Im}\,K(\eta_1, k) + (\eta_1 \leftrightarrow \eta_2)\right]$$



Symmetries

- Cosmological perturbations are observed to be statistically *homogeneous* and *isotropic*
- Primordial perturbations are also observed to be approximately scale invariant
- Anything else?
  - With *de Sitter boost* we can derive general results and connect with Conformal Field Theory and holography. See beautiful progress by Maldacena, Pimentel, Arkani-Hamed, Baumann, Joyce, etc…
  - If we are instead more interested in phenomenology, we cannot assume Boost invariance.



Boost/ess theories

- All cosmological models break Lorentz/de Sitter boosts. *The breaking of boosts can be large and is NOT slow-roll suppressed*, in contrast to the small breaking of dilation

assumed observed symmetries

$$\sum_{a=1}^{n} \vec{k}_a \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ translations

$$\sum_{a=1}^{n} k_a^{[i} \partial_{k_a^{j]}} \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ rotations

$$\sum_{a=1}^{n} \left(3 - \Delta + k_a \partial_{k_a}\right) \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ dilations

$$\sum_{a=1}^{n} \left[2\vec{k} \cdot \vec{\partial}\partial_i - k_i \partial^2 + 2\left(3 - \Delta\right) \partial_i\right] \langle \phi(k_1) \dots \phi(k_n) \rangle = 0$$ dS boosts

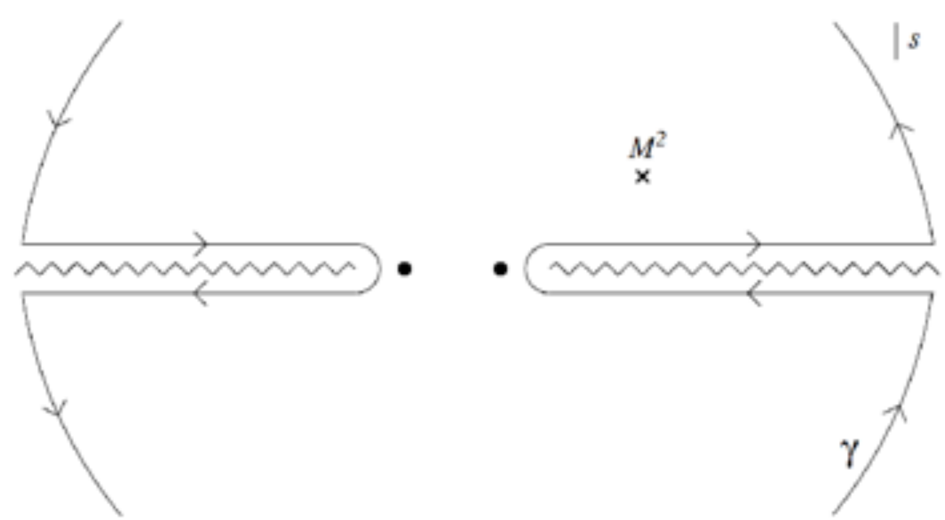
additional

The Analytic Wavefunction

Analyticity and causality

- There is a well-known connection between causality and analyticity, which leads to powerful UV/IR sum rules, analogous to the Kramers Kronig “dispersion relation”.

- Ex: operators commuting outside the light cone implies the 2-to-2 amplitude is analytic in Mandelstam s at fixed t



- What is the analytic structure of wavefunction coefficients?

- Here are some results [Goodhew, Lee, Melville & Pajer ’22]

Off-shell wavefunction

- Analytic in what?! We need to go off-shell.

- Off-shell wavefunction coefficients are the F-transform of amputated (i.e. acted on eq. of motions for all fields), connected in-out Green's functions

$$\psi_n(\{\omega\},\{\mathbf{k}\}) = \left[\prod_{j=1}^{n}\int_{-\infty}^{0} d\eta_j\, K_\nu(\omega_a,\eta_a)\right] G^{\text{amp.con.}}_{\mathbf{k}_1\ldots\mathbf{k}_n}(t_1,\ldots,t_n)$$

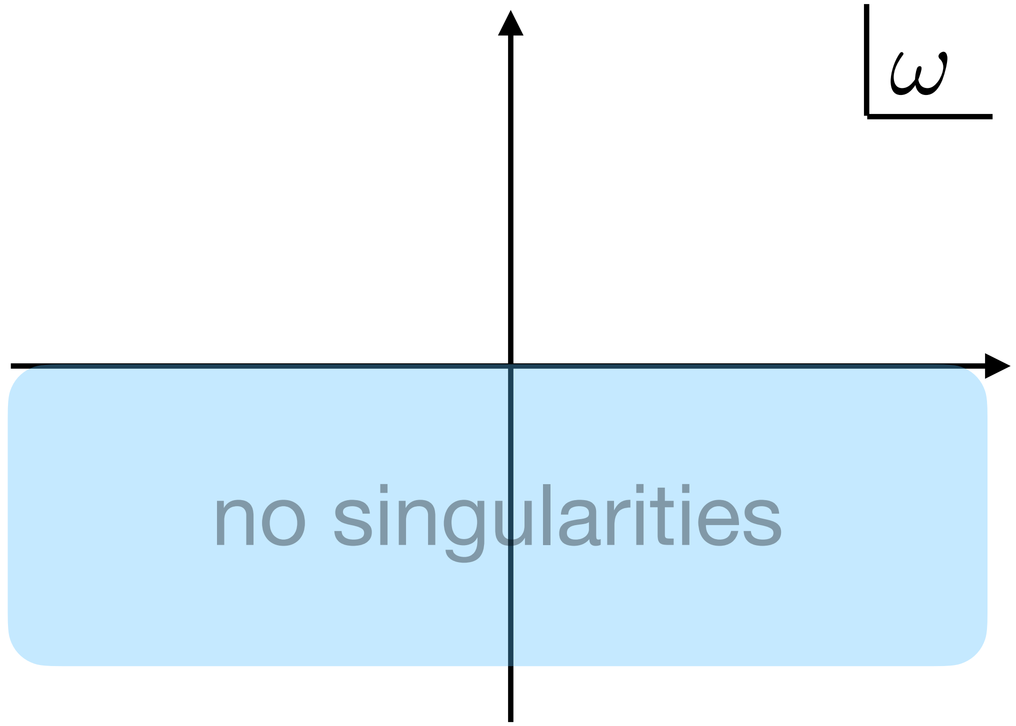
$$\langle\phi(0)=0|\, T\prod_a^n \hat{\phi}_{\mathbf{k}}(t_a)|\Omega_{\text{in}}\rangle_{con} = G_{\mathbf{k}_1\ldots\mathbf{k}_n}(t_1,\ldots,t_n)\,\delta_D^{(3)}\left(\sum_a^n \mathbf{k}_a\right)$$

- where K are mode functions in any FLRW spacetime with Bunch-Davies initial conditions.

- This is valid non-perturbatively. Reminiscent of LSZ.

Analyticity

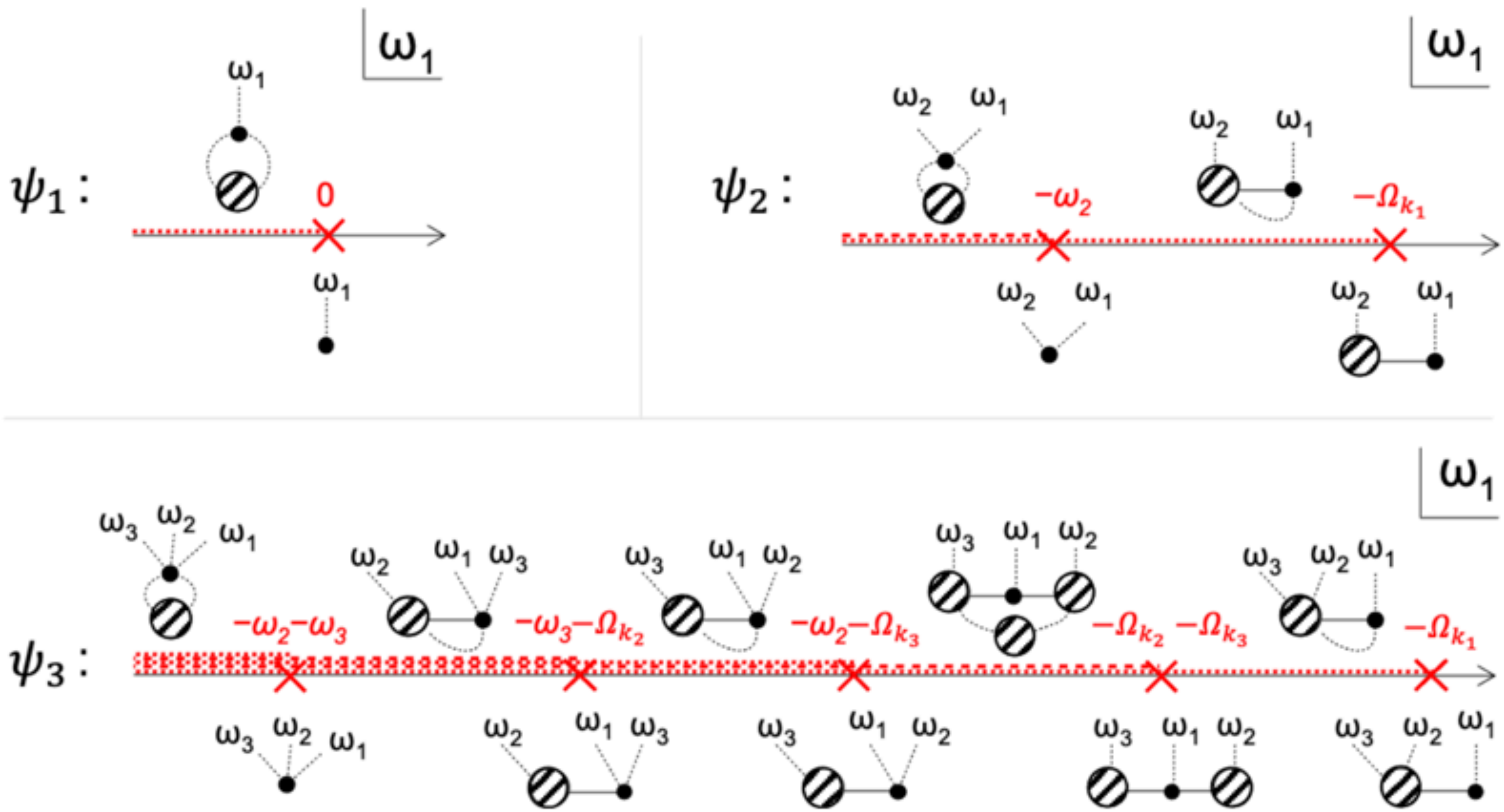
$$\psi_n(\omega) \sim \int_{-\infty}^{0} dt\, e^{i\omega t}\, G(t)$$



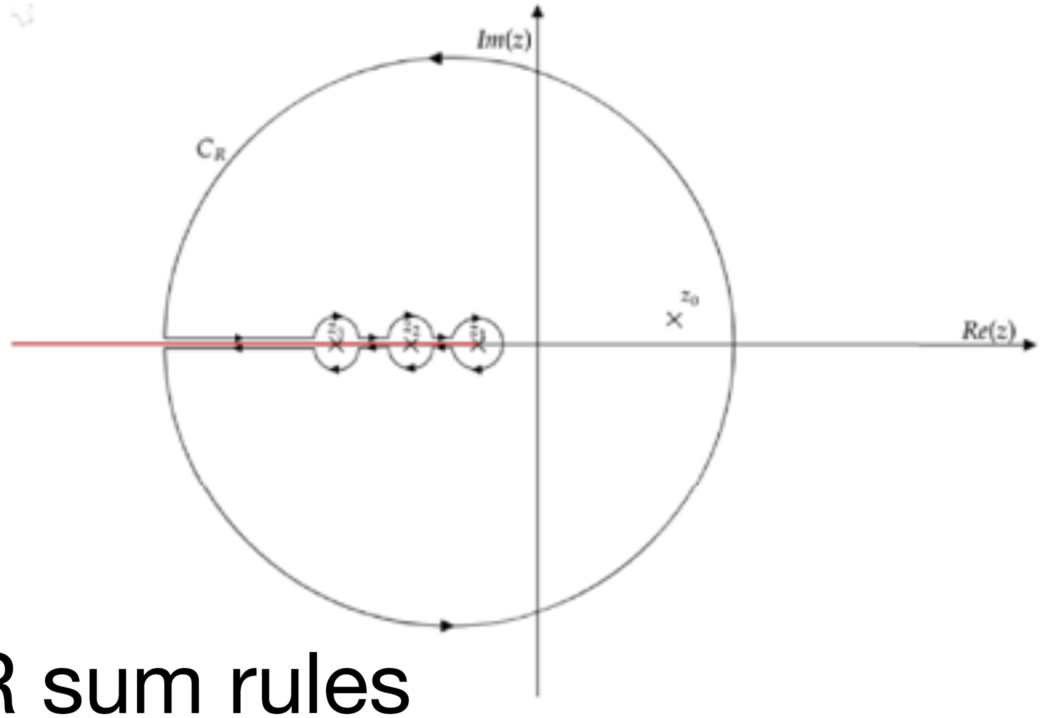
- Time integral for $\Psi[\phi,\eta_0]$ stops at η_0 because of causality
- Then ψ_n are analytic in ω in the lower-half complex plane because the integral is even more convergent. This is true non-perturbatively
- Extend to upper-half plane by Hermitian analyticity $\psi_n(\omega^*)=\psi_n^*(\omega)$
- Singularities only on the real axis

All singularities

- Consider off-shell coefficients as function of a single ω with other kinematics fixed. Singularities occur only on the negative real ω axis where the energy of a perturbative subdiagram vanishes. (There might be additional anomalous thresholds)



UV/IR sum rules

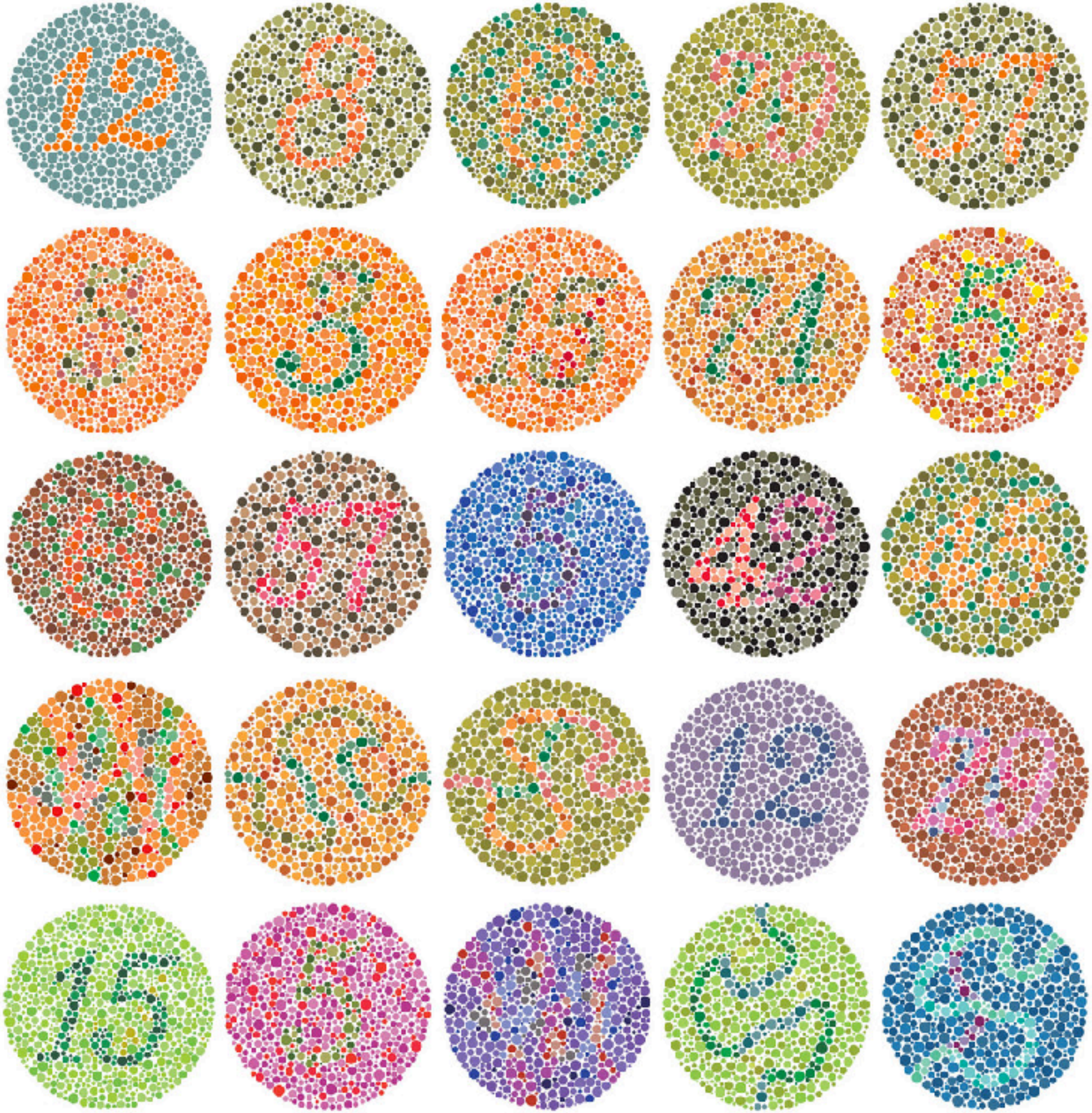


- By Cauchy's theorem we can write UV/IR sum rules

$$\omega_T \psi_{EFT}(\omega_i, \mathbf{k}_j) = \int_{-\infty}^{0} \frac{d\omega}{2\pi i} \frac{\mathrm{disc}(\omega_T \psi_{\mathrm{UV}}(\omega, \omega_{i\neq 1}, \mathbf{k}_j))}{\omega - \omega_1} + \underset{\infty}{\mathrm{Res}}\left(\frac{\omega_T \psi_{\mathrm{UV}}(\omega, \omega_{i\neq 1}, \mathbf{k}_j)}{\omega - \omega_1}\right).$$

- The LHS can computed in a low-energy EFT. The RHS depends on the full UV theory.

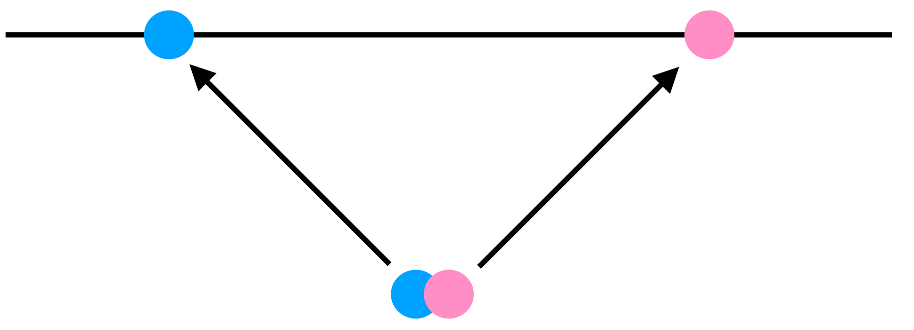
- This fixes all Wilson coefficients in the EFT, including total derivatives and terms proportional to the eq of motion.



Locality

Locality

- Locality: what happens here cannot affect what happens far away. Operators commute for space-like separation and correlators factorise at large distances (*cluster decomposition*).



- There is no cluster decomposition in dS

- A common sufficient condition is *Manifest Locality*: Lagrangian interactions are products of operators *at the same spacetime point*. No inverse laplacians are allowed.

- The wavefunction of light *scalars and spin-2 fields* ($m^2 < 2H^2$) satisfies *non-perturbatively* the *Manifestly Local Test (MLT)* [Jazayeri, EP & Stefanyszyn '21]

$$\partial_{\omega_1} \psi_n(\omega_1, \ldots, \omega_n; \mathbf{k}_1, \ldots, \mathbf{k}_n)\Big|_{\omega_1=0} = 0$$

manifest locality ->
$$\partial_{\omega_1} \psi_n(\omega_1, \ldots, \omega_n; \mathbf{k}_1, \ldots, \mathbf{k}_n)\Big|_{\substack{\omega_1=k_1=0 \\ \omega_a=|\mathbf{k}_a|}} = 0$$

Derivation

- Two derivations: (*i*) boundary derivation using unitarity and singularities (see paper) and (*ii*) a bulk derivation that uses

$$\psi_n\left(\{\omega\},\{\mathbf{k}\}\right)=\left[\prod_{a=1}^{n}\int_{-\infty}^{0}d\eta_a K(\omega_a\eta_a)\right]G^{\text{amp.}}_{\mathbf{k}_1\ldots\mathbf{k}_n}(t_1,...,t_n)$$

- Notice that as k->0 there is no linear term in K

$$\partial_\omega K_{3/2}(\omega,\eta)|_{\omega=0}=\partial_\omega(1-i\omega\eta)e^{i\omega\eta}|_{\omega=0}=0$$

- The same is true non-perturbatively for $\psi_n(\omega)$

- By manifest locality we can take this on-shell *ω*=*k* and it is still 0 because there cannot be a divergence at k=0

Manifest locality

- The Manifestly Local Test is a *necessary* condition for all manifestly local test. All large non-Gaussianities in single field inflation (e.g. EFT of inflation) obey this

$$\mathcal{L} \supset \dot{\phi}^3 + (\partial\phi)^2\dot{\phi} + \dot{\phi}^4 + \ldots$$

- Gravity has non-manifestly-local interactions for backreacting scalars after integrating out lapse and shift

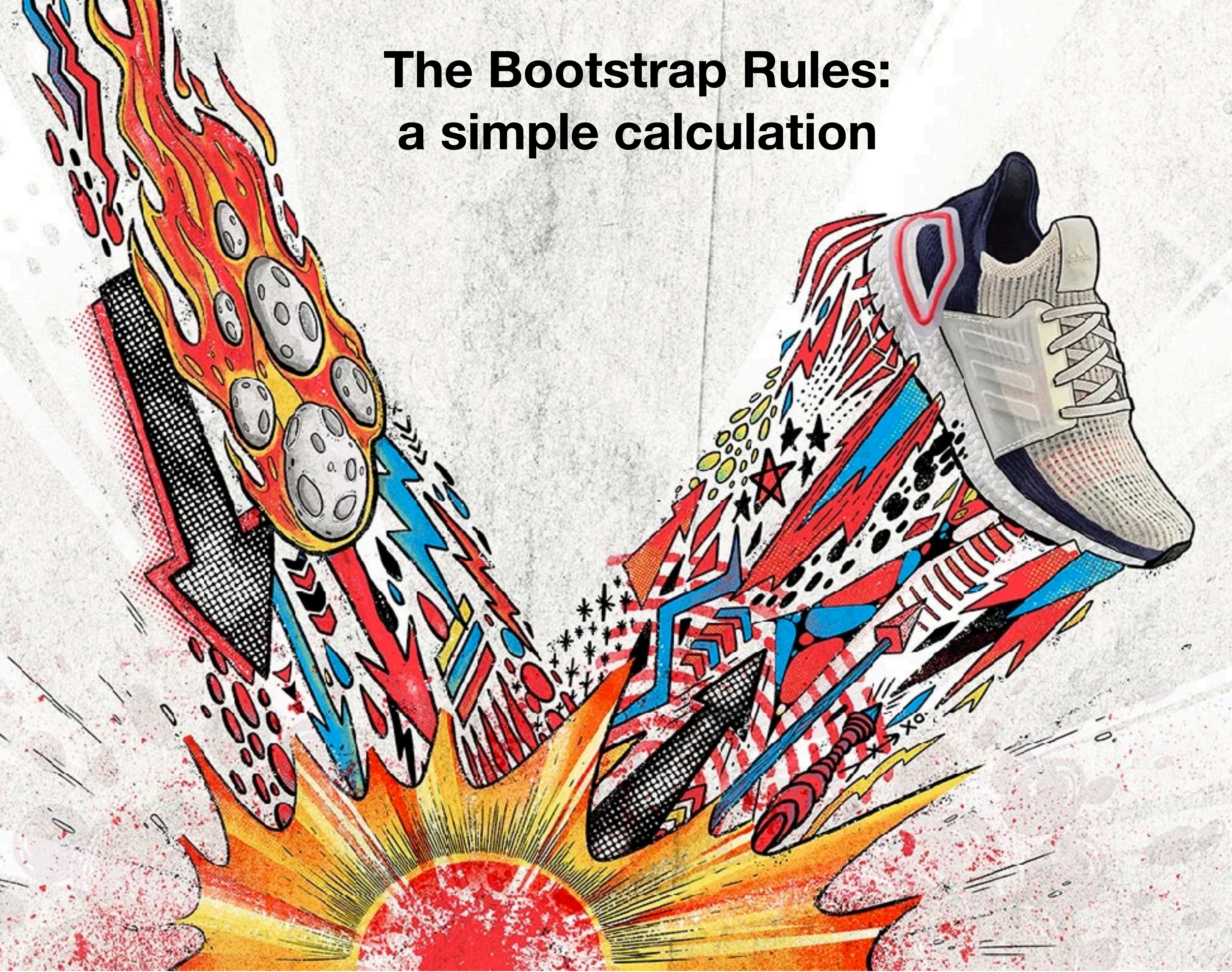
$$\mathcal{L}_{GR} \supset \dot{\zeta}^2\nabla^{-2}\dot{\zeta} + \ldots$$

But the MLT applies more generally to all "soft" interactions

$$\mathcal{L} \supset \int_{\{\mathbf{k}\}} \prod_a^n \phi(\mathbf{k}_a) F(\{\mathbf{k}\}) \;\; \text{s.t.} \;\; \partial_{\mathbf{k}_a} F(\{\mathbf{k}\})\Big|_{\mathbf{k}_a=0} = 0$$

In particular, the *MLT applies to gravitons and spectator fields to all orders in perturbation theory.*

The Bootstrap Rules: a simple calculation



Bootstrap Rules

- Instead of computing bispectra from a model we use a set of Bootstrap Rules based on fundamental principles [EP '20]

- As an example, let's bootstrap the bispectrum (3-point function) of a *scalar.* We can work directly on shell wlog.

- It can only have kT poles by locality!

Scale invariance

Bose symmetry

$$\psi_3 = \frac{\mathrm{Poly}_{3+p}(\overbrace{k_T, e_2, e_3})}{k_T^p}$$

$$k_T \equiv k_1 + k_2 + k_3$$

$$e_2 \equiv k_1 k_2 + k_2 k_3 + k_1 k_3$$

$$e_3 \equiv k_1 k_2 k_3$$

tree level in dS

Bunch Davies vacuum

The calculation

- The Bootstrap Rules reduced the problem to determining the numerical constants C_{mn} via the Manifestly Local Test

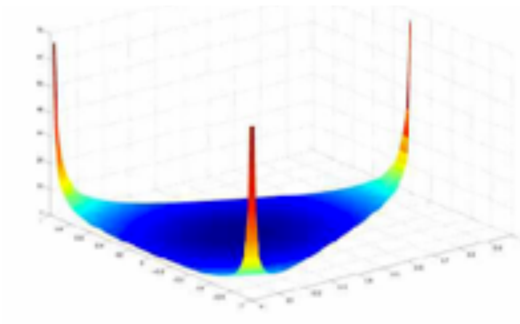
$$\psi_3^{(p)}(k_1,k_2,k_3) = \frac{1}{k_T^p} \sum_{n=0}^{\lfloor \frac{p+3}{3} \rfloor} \sum_{m=0}^{\lfloor \frac{p+3-3n}{2} \rfloor} C_{mn} k_T^{3+p-2m-3n} e_2^m e_3^n,$$

$$\partial_{k_1} \psi_3 \Big|_{k_1=0} = 0$$

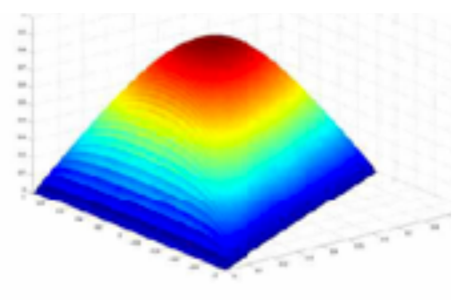
- This yields all manifestly local bispectra for a scalar to *any order in derivatives* in the EFT of inflation

- This gives order by order the shapes of non-Gaussianity that are constraint e.g. by the Cosmic Microwave Background, e.g. the Planck mission

Shapes of non-Gaussianity



$$\psi_3^{(0)} = A_0 \left[4e_3 - e_2 k_T + (3e_3 - 3e_2 k_T + k_T^3)\log(-k_T \eta/\mu)\right]$$
$$\psi_3^{(1)} = 0$$
$$\psi_3^{(2)} = A_2 \left[-k_T^3 + 3k_T e_2 - 11 e_3 + \frac{4e_2^2}{k_T} + \frac{4e_2 e_3}{k_T^2}\right]$$
$$\psi_3^{(3)} = A_3 \frac{1}{k_T^3}(k_T^6 - 3k_T^4 e_2 + 11 k_T^3 e_3 - 4k_T^2 e_2^2 - 4k_T e_2 e_3 + 12 e_3^2) + A_3' \frac{e_3^2}{k_T^3}$$



- $\Psi_3^{(0)}$ contains the famous *local non-Gaussianity,* while $\Psi_3^{(1,2)}$ the so-called *equilateral and orthogonal non-Gaussianities*, the main targets of non-Gaussian searches in the CMB and galaxy surveys!

- In the standard approach the numerical coefficient come from time integrations, here they're fixed algebraically


Unitarity

Unitary time evolution

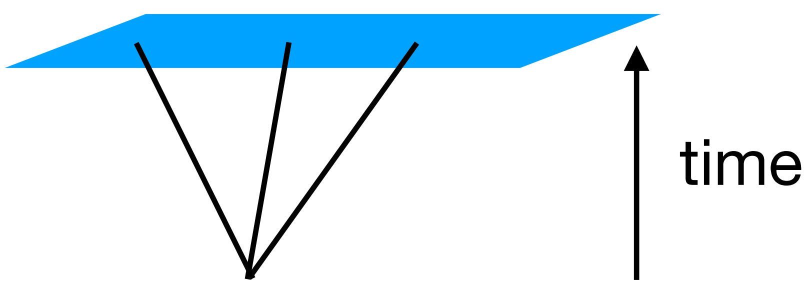
- In Quantum Mechanics we compute probabilities, which must be between 0 and 1 to make sense
- This requires the positive norm of states in the Hilbert space *and* Unitary time evolution, $UU^\dagger=1$. Colloquially this is the *conservation of probabilities*
- The consequences of unitarity for particle physics amplitudes were discover over *60 years ago*: the Optical theorem and Cutkosky Cutting Rules.
- In cosmology we don’t see the time evolution, so how can we see it’s unitary?!

The Cosmological Optical Theorem (COT) [Goodhew, Jazayeri, EP ’20]

- From unitarity, $UU^{\dagger}=1$, we found infinitely many relations.

- The simplest applies to contact n-point functions

$$\psi_n(\{\omega\},\{\mathbf{k}\})+\psi_n^*(\{-\omega\},\{-\mathbf{k}\})=0$$



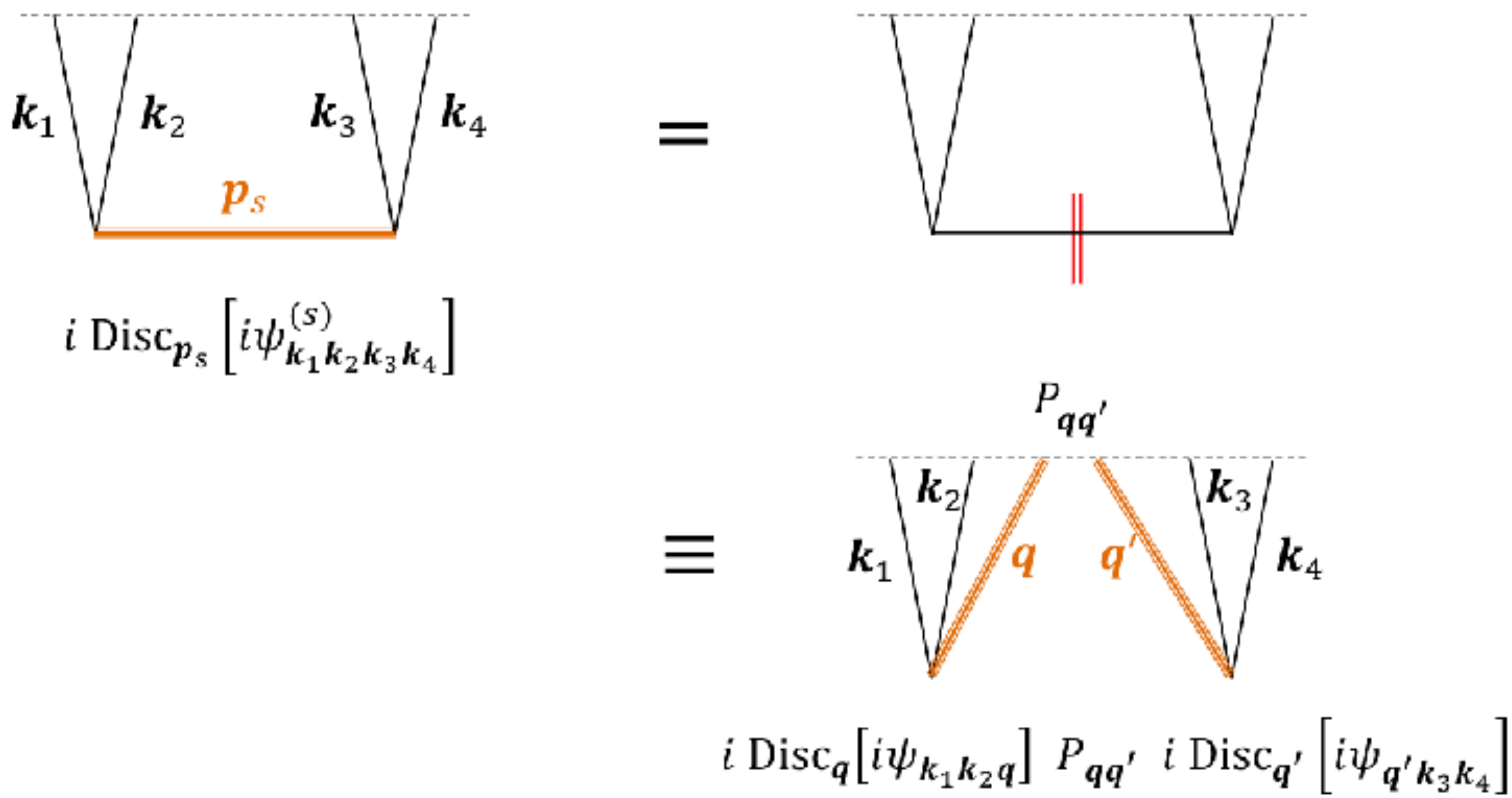
- It follows from unitarity time evolution, but the equation does not involve time! Time “emerges” at boundary as in holography…

- This is a Cosmological Optical Theorem (COT) and can be interpreted as fixing a “discontinuity”

$$\mathrm{Disc}\psi_n=\psi_n(\{\omega\},\{\mathbf{k}\})+\psi_n^*(\{-\omega\},\{-\mathbf{k}\})=0$$

Exchange diagrams

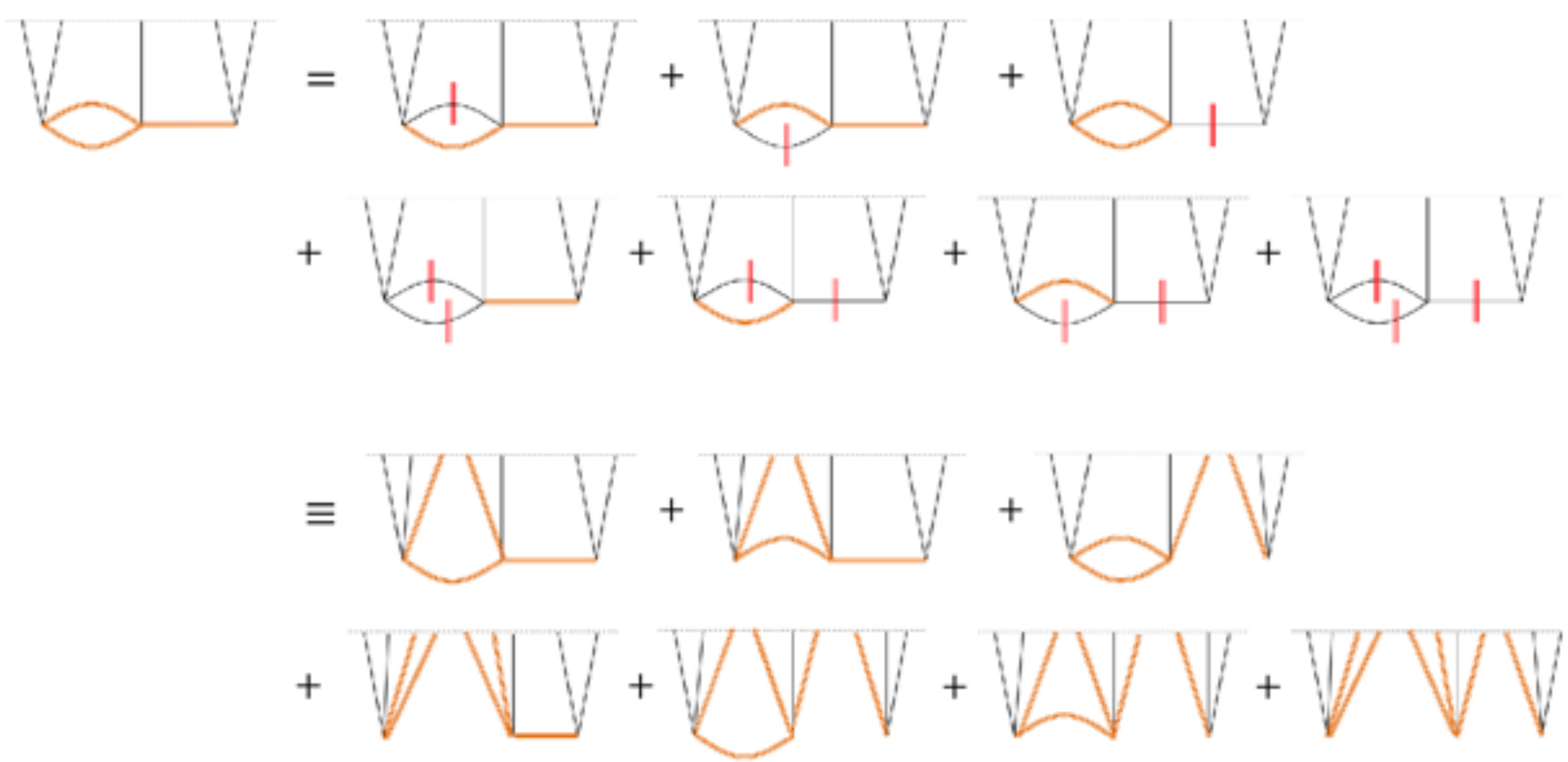
- The next simplest case is a 4-particle exchange diagram (trispectrum). The Cosmo Optical Theorem (COT) is



General diagrams

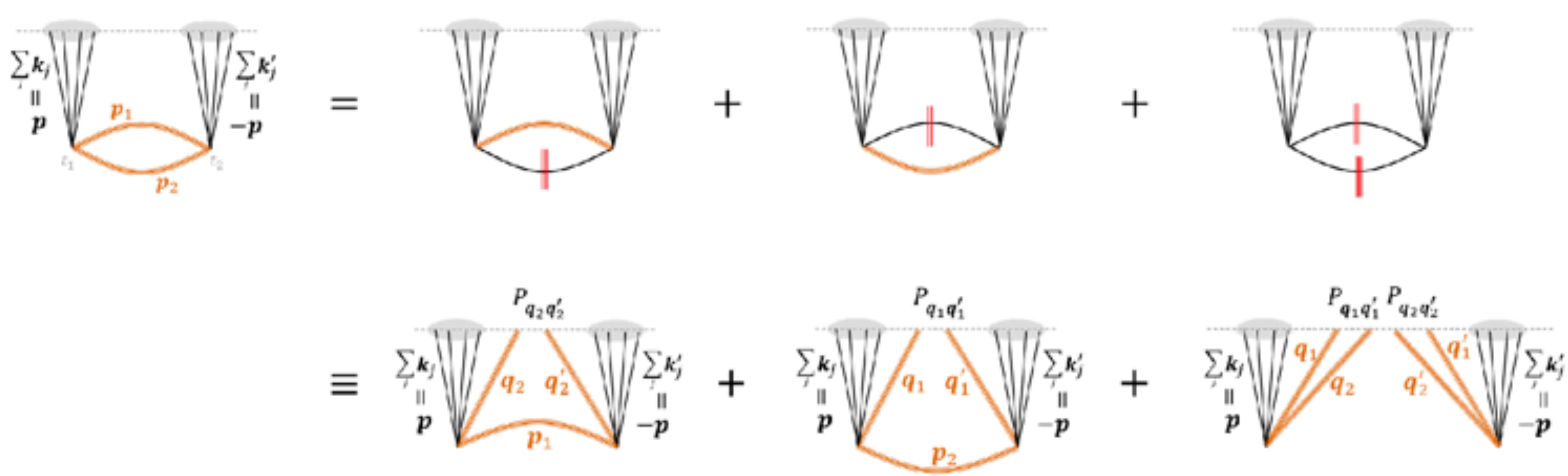
- These relations are valid to *all order in perturbation theory to any number of loops for fields of any mass and spin and arbitrary interactions (around any FLRW admitting a Bunch Davies initial condition)* [Goodhew, Jazayeri & EP '21; Melville & EP '21]

$$i \underset{\substack{\text{internal}\\ \text{lines}}}{\text{disc}} \left[i\,\psi^{(D)} \right] = \sum_{\text{cuts}} \left[\prod_{\substack{\text{cut}\\ \text{momenta}}} \int P \right] \prod_{\text{subdiagrams}} (-i) \underset{\substack{\text{internal \&}\\ \text{cut lines}}}{\text{disc}} \left[i\,\psi^{(\text{subdiagram})} \right] ,$$

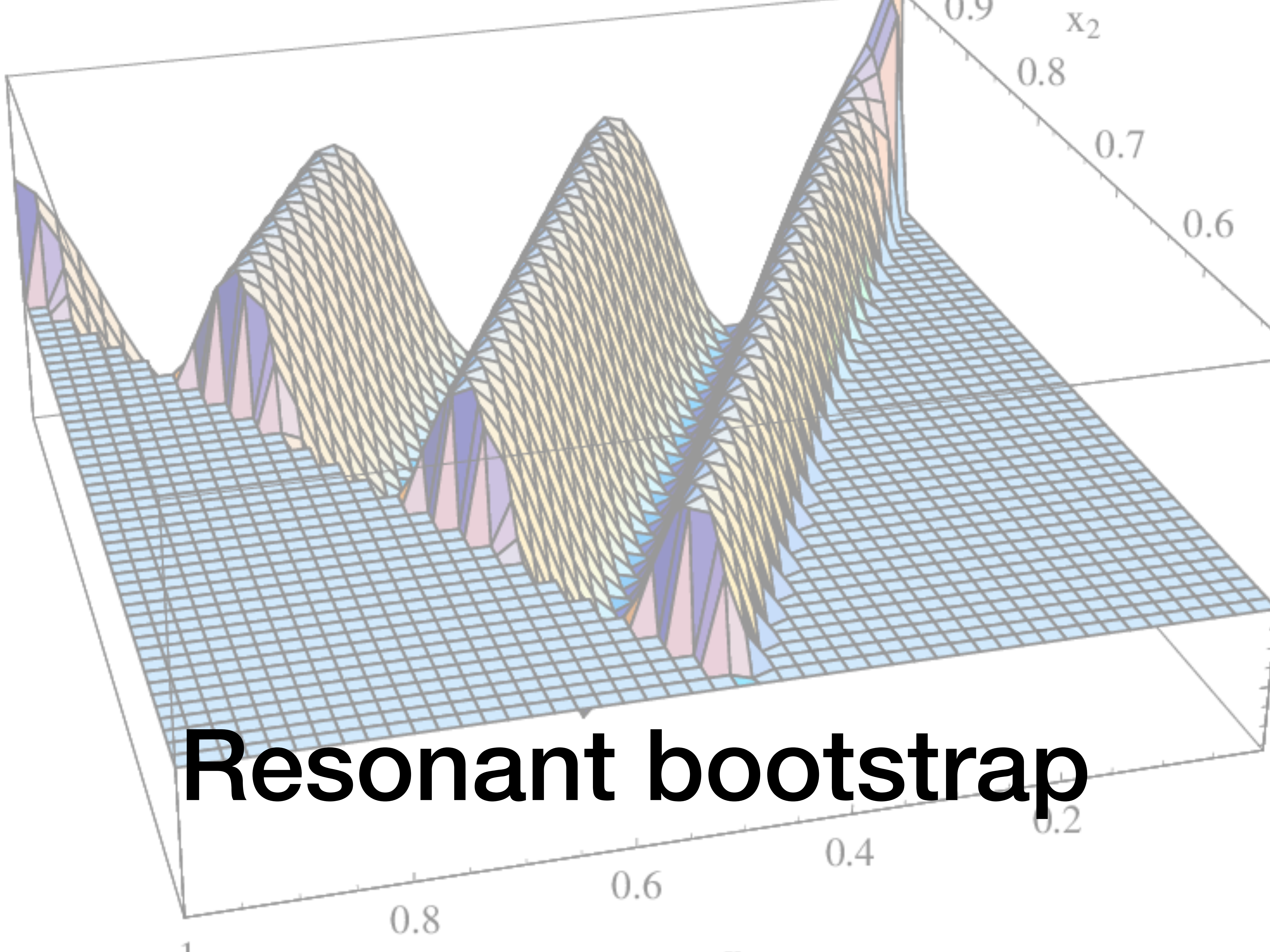


Loop corrections

- Unitarity gives us also *loop corrections*! For example we compute the leading 1-loop corrections for the power spectrum in the EFT of inflation, from tree-level results.



$$i\text{Disc}\left[i\psi^{\text{1-loop}}_{\mathbf{k}_1\mathbf{k}_2}\right] = \frac{H^2}{f_\pi^4}\frac{ik^3}{480\pi}\frac{(1-c_s^2)^2}{c_s^4}\left[(4\tilde{c}_3+9+6c_s^2)^2+15^2\right]$$


Resonant bootstrap
x_2
0.8
0.7
0.6
0.2
0.4
0.6
0.8

Beyond scale invariance

- The assumptions of scale invariance greatly simplify the derivation of correlators/wavefunction using the cosmological bootstrap, but it is not necessary

- Example beyond scale invariance: periodic *oscillations* in time, as in axion (monodromy) induced by gauge instantons

- Scale invariance is broken to a discrete subgroup

$$\exists\, \lambda \in \mathbb{R} : \quad \psi_n(\lambda^m \omega, \lambda^m \mathbf{k}) = \lambda^{3m} \psi_n(\omega, \mathbf{k}) \quad \forall m \in \mathbb{N}$$

- Most general local, tree-level, on shell ansatz is $\alpha = 2\pi / \log \lambda$

$$\psi_3(k_a) = \left[\left(\frac{i\alpha}{k_T} \right)^p \mathrm{Poly}^{(p)}_{p+3}(k_a) + \left(\frac{i\alpha}{k_T} \right)^{p-1} \mathrm{Poly}^{(p-1)}_{p+2}(k_a) \right] e^{-i\alpha \log{(k_T/k_*)}} + O(\alpha^{p-2}) \,.$$

Resonant bootstrap

- Resonant ψ_3 obeys the cosmo optical theorem (COT), but this requires exponentially suppressed contributions (from the stationary phase solution for the oscillating integral)

- Instead we use a modified COT for contact diagrams

$$\psi_3(k_a, \alpha) + \psi_3^*(-k_a, -\alpha) = 0$$

- the coefficients of the polynomials are all real by unitarity

- The polynomial are fixed by the manifestly-local test to all orders in derivatives.

Quantum interference

- To leading order we reproduce resonant non-Gaussianity (ϕ³)

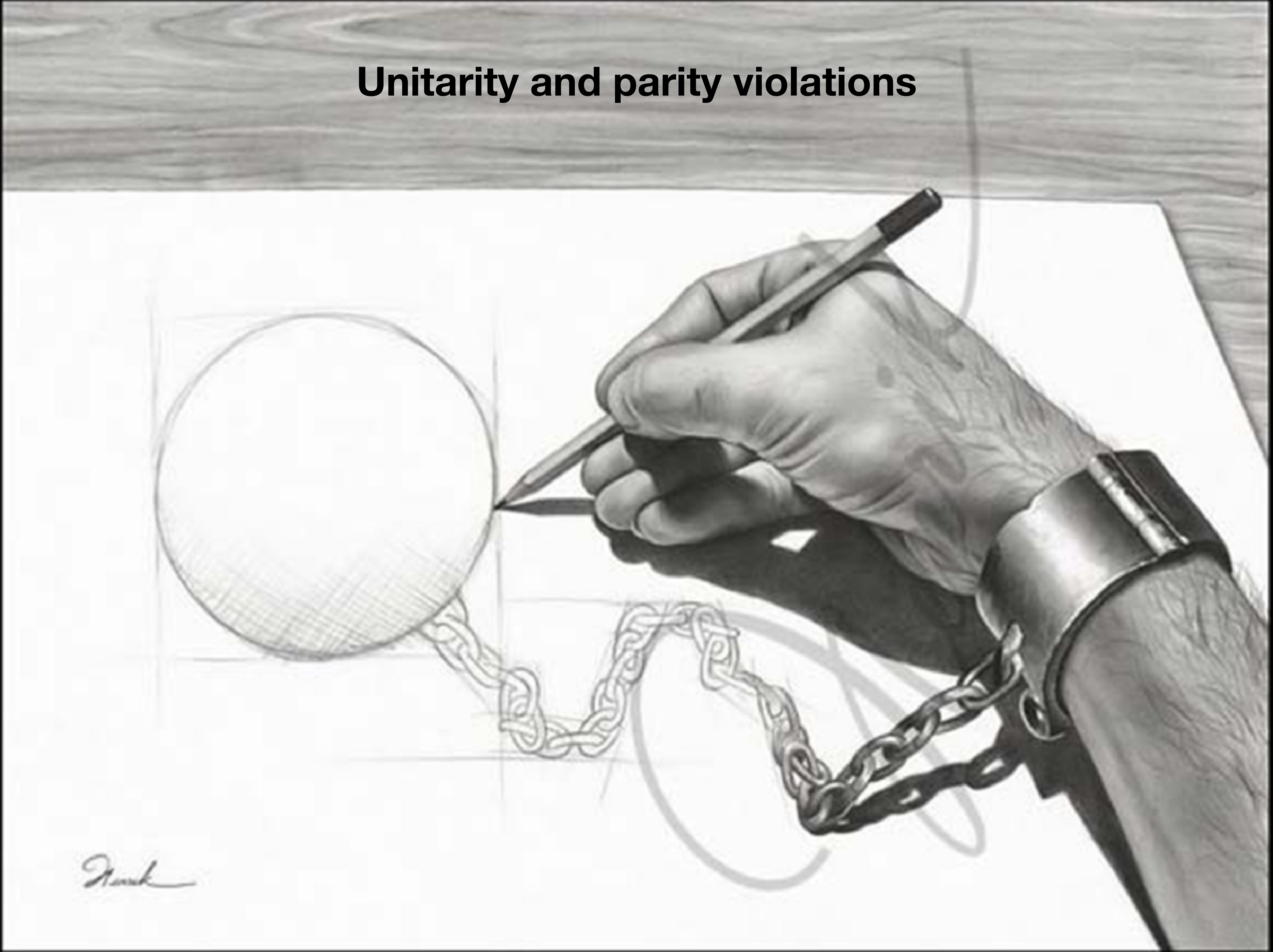
$$\psi_3^{(0)} = C_{0,0}\left(e_3 - i\frac{k_T e_2}{\alpha}\right) e^{-i\alpha \log{(k_T/k_*)}} \,.$$

- The real and imaginary part of the wavefunction interfere with each other when computing the bispectrum

$$B_{\pi^3} \propto \frac{1}{e_3^2}\left[\cos\Big(\alpha\log(k_T/k_*)\Big) - \frac{1}{\alpha}\sum_{i,j}\frac{k_i}{k_j}\,\sin\Big(\alpha\log(k_T/k_*)\Big)\right]$$

- Subleading out-of-phase oscillations are a quantum effect

Unitarity and parity violations



Contact interactions

- Contact interactions contribute to correlators as

$$\langle \varphi(\mathbf{k}_1)\ldots\varphi(\mathbf{k}_n)\rangle = \frac{\int \mathcal{D}\varphi\ \Psi\Psi^*\ \varphi(\mathbf{k}_1)\ldots\varphi(\mathbf{k}_n)}{\int \mathcal{D}\varphi\ \Psi\Psi^*}\,,$$

$$B_n^{\text{contact}}(\{k\};\{\mathbf{k}\}) = -\frac{\psi_n'(\{k\};\{\mathbf{k}\}) + \psi_n'^*(\{k\};-\{\mathbf{k}\})}{\prod_{a=1}^n 2\ \text{Re}\ \psi_2'(k_a)}\,,$$

which gives the Real or Imaginary part for parity even or odd interactions.

$$\psi_n(\{\omega\},\{\mathbf{k}\})$$

parity

$$\pm\psi_n(\{\omega\},\{-\mathbf{k}\})$$

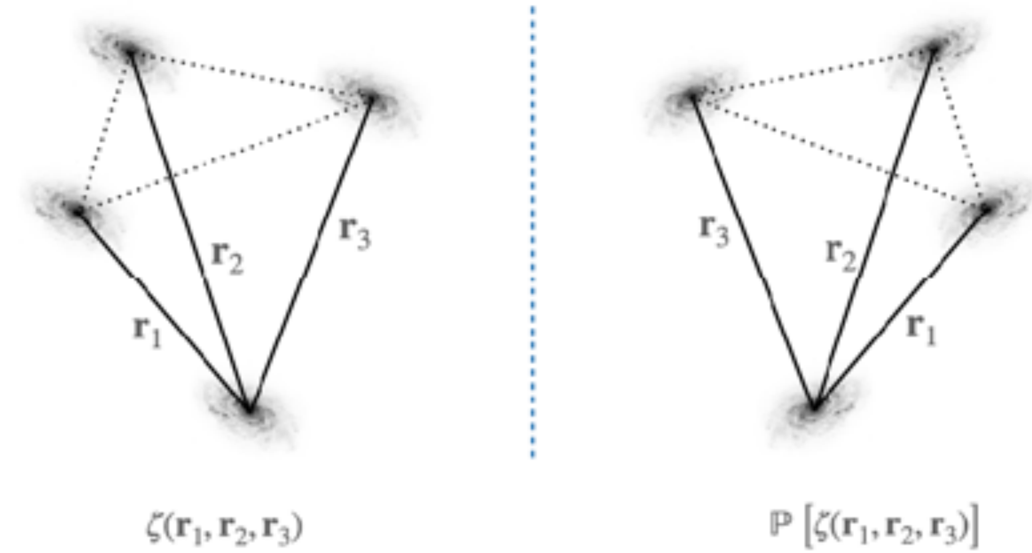
scaling

$$\lambda^{-3}\psi_n(\{\lambda\omega\},\{\lambda\mathbf{k}\})$$

cosmo optical theorem

$$\psi_n^*(\{-\omega\},\{-\mathbf{k}\})$$

No-go for parity odd



- Assuming scale invariance, unitarity and a BD initial state, *IR-finite* parity-odd correlators vanish for [Cabass, Jazayeri, EP & Stefanyszyn '22]
    - Any number of external massless field interacting with conformally coupled scalar fields
    - The trispectrum of a massless scalar at tree-level interaction with any number of massive scalars, or massless fields of any spin.
- If the recent hints [Cahn, Slepian & You '22; Philcox '22] of a parity odd galaxy trispectrum were confirmed, one would need to relax these assumptions, e.g. break scale invariance, massive or chiral spinning fields

Summary

- For massless scalars and gravitons, locality, unitarity and scale invariance imply that:
  - parity-odd contact correlators, such as the tree-level bispectrum, cannot have kT-poles and come in a small number, even though there are infinitely many interactions in the Lagrangian!
  - Tree-level GR correlators are rational functions (in even spacetime dimensions). No logs or polylogs.

All Graviton non-Gaussianities

All graviton bispectra

- All tree-level, scale-invariant graviton bispectra on de Sitter must take the form

$$\psi_3^{+++} = \sum \left[e^+(\mathbf{k}_1) e^+(\mathbf{k}_2) e^+(\mathbf{k}_3) \mathbf{k}_1^{\alpha_1} \mathbf{k}_2^{\alpha_2} \mathbf{k}_3^{\alpha_3} \right] \psi_3^{\rm trim}(k_1, k_2, k_3) \,,$$

- In terms of spinors this is (all other polarizations are fixed by this)

$$I_i = k_T - 2k_i$$

$$\psi_3^{+++} = \frac{[12]^2[23]^2[31]^2}{e_3^2} \sum_{\rm perm's} h_\alpha(k_1, k_2, k_3) \psi_3^{\rm trim}(k_1, k_2, k_3) \,,$$

$$h_0 = 1 \,, \quad h_1 = ik_1 \,, \quad h_2 = k_2 k_3 \,, \quad h_3 = iI_1 I_2 I_3 \,, \quad h_4 = I_1^2 I_2 I_3 \,,$$

$$h_{5a,b} = iI_1^3 I_2 I_3, iI_1 I_2^2 I_3^2 \,, \quad h_6 = I_1^2 I_2^2 I_3^2 \,, \quad h_7 = iI_1^3 I_2^2 I_3^2 \,.$$

- The trimmed wavefunction is the most general solution to the Manifestly Local Test (MLT)

Parity-odd graviton bispectra

- *Parity-odd* graviton non-Gaussianity is a poster child of the *boostless* cosmo bootstrap. The are infinitely many interactions

$$L \supset \cdots + \epsilon^{\mu\nu\rho\sigma} R_{\rho\sigma\lambda\delta} R^{\lambda\delta}{}_{\gamma\theta} R^{\gamma\theta}{}_{\mu\nu} R^n + \ldots$$

- Yet, *to all orders in derivatives* there are only *three* possible bispectra (notice no kT poles)!

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{k_T \left(k_T^2 - 2e_2\right)}{e_3^3} \, ,$$

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{(-3e_3 + k_T e_2)}{e_3^3} \, ,$$

$$B_3^{+++} = e^+_{ij}(\mathbf{k}_1) e^+_{jk}(\mathbf{k}_2) e^+_{ik}(\mathbf{k}_3) \frac{(-k_1 + k_2 + k_3)(k_1 - k_2 + k_3)(k_1 + k_2 - k_3)}{e_3^3} \, .$$

Parity-odd mixed bispectra

- Similarly, parity-odd Scalar-Scalar-Tensor and Scalar-Tensor-Tensor bispectra to all orders in derivates can only be:

$$B_3^{00+} = \frac{[13]^2[23]^2}{k_3^2[12]^2}\frac{(k_1+k_2-k_3)^2 k_3}{e_3^3}\,,$$

$$B_3^{0++} = \frac{[23]^4}{k_2^2k_3^2e_3^3}[q_{1,1}(k_2+k_3)k_1^2 + q_{1,2,a}(k_2^3+k_3^3) + q_{1,2,b}(k_2k_3^2+k_3k_2^2)]\,,$$

$$B_3^{0+-} = \frac{[12]^4}{[31]^4}\frac{I_2^4}{k_2^2k_3^2e_3^3}\left[q_{1,1}(k_2-k_3)k_1^2 + q_{1,2,a}(k_2^3-k_3^3) + q_{1,2,b}(k_2k_3^2-k_3k_2^2)\right]\,,$$

Parity-even bispectra

- There are infinitely many parity-even graviton bispectra, corresponding to interactions with a larger and larger number of derivatives

- In a technically natural theory, the larger the number of derivatives, i.e. the order of the polynomial, the smaller the contribution should be.

$$B_3^{+++} = \frac{\mathrm{SH}_{+++}}{e_3^3}\Bigg[g_{0,0}\left(4e_3 - e_2 k_T + (k_T^3 - 3k_T e_2 + 3e_3)\log(-k_T\eta_0/\mu)\right) + g_{0,2}\frac{e_2 e_3 + e_2^2 k_T - 2e_3 k_T^2}{k_T^2} + g_{0,3}\frac{e_3^2}{k_T^3} + \dots \Bigg],$$

Phenomenology

Perturbativity and the bounds on the tensor-to-scalar ratio imply various phenomenological constraints:

- Since there are only a few parity-odd shape, they should be a *primary targets* of observations (mostly CMB B-modes)
- Graviton bispectra have a smaller signal-to-noise ratio (S/N) than the graviton power spectrum, so they can be seen only after a detection of primordial tensor modes
- <scalar^2 tensor> has always a larger S/N than <tensor^3> or <scalar tensor^2> by a factor of ϵ^{-1}, so it should be the *first target,* unless one probes only the tensor sector

Horizons

- There are still basic and very general facts about quantum field theory on cosmological spacetimes that are awaiting to be discovered: *it's a wide open field of research*!
- Questions for the future include:
  - Can we derive "positivity bounds" for cosmology that encode the constraints of a consistent UV completion?
  - Are there measurable non-perturbative quantum gravity effects in cosmological correlators as e.g. in Black Hole physics?
  - Numerically bootstrap fully non-perturbative correlators in dS?
- Because of the ever growing body of cosmological dataset, advancements on the theory side are likely to have important repercussion on the phenomenology and ultimately make a long standing contribution to our understanding of the very early universe.