Diverging black hole entropy from quantum infrared non-localities

Alessia Platania

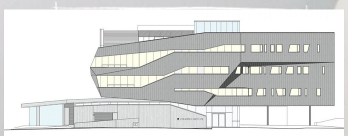
Based on:

A. Platania, J. Redondo-Yuste - arXiv:2303.17621

Quantum gravity and cosmology 2023

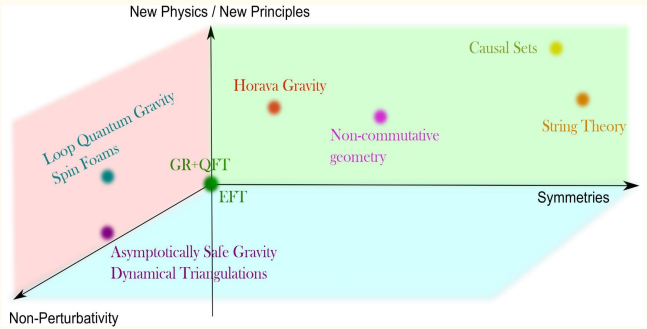
19.04.2023

PERIMETER INSTITUTE



Appetizer: outline & spoiler

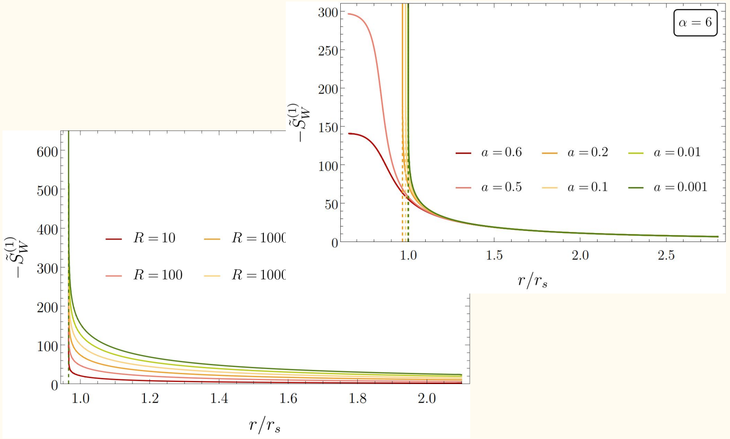
- Different approaches to quantum gravity



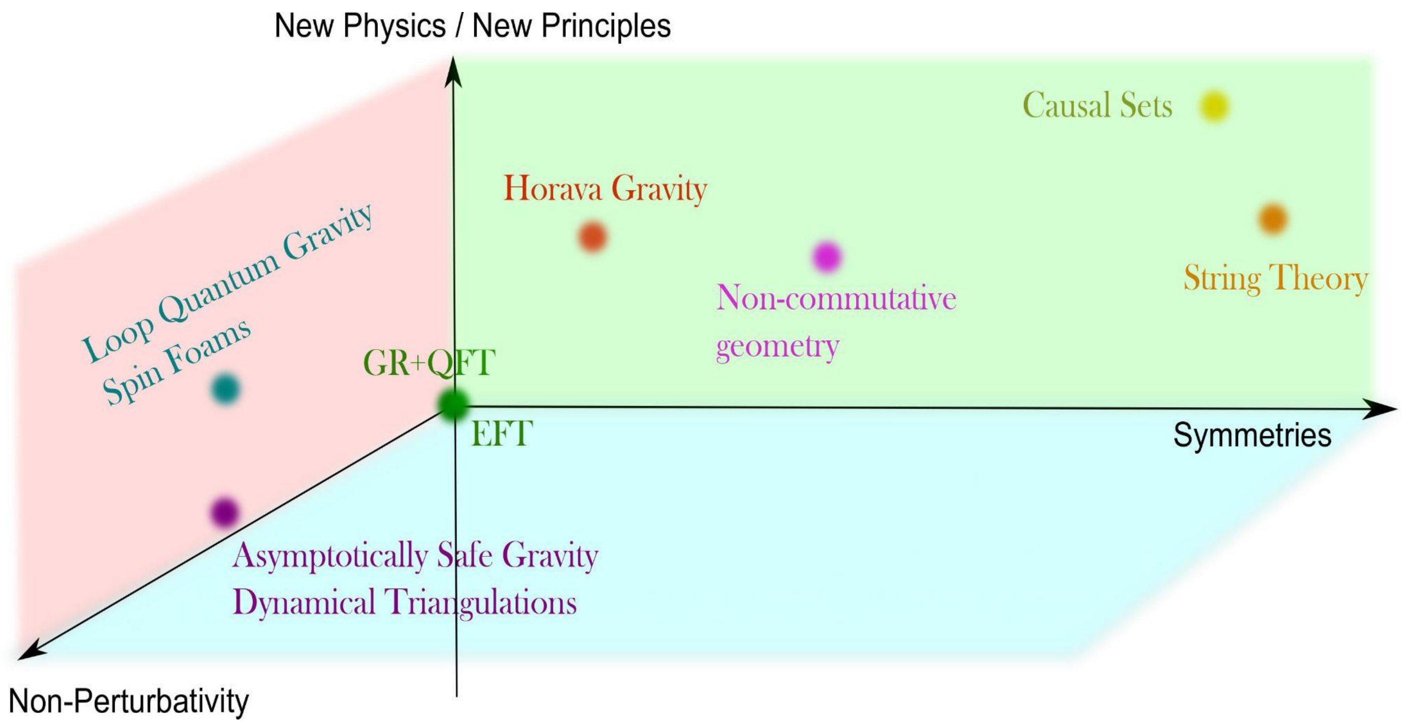
- Effective actions from different approaches
  ⇒ different effective non-localities

- IR non-localities in effective actions:
  - Phenomenologically appealing
  - Occur in various QG models

- Modified black hole entropy?
  Result: generally divergent for large black holes, modulo fine tuning
  ⇒ non-trivial test for pheno and QG models



The realm of Quantum Gravity



Effective Actions in Quantum Gravity

- **Goal: predictions, contrast different theories, put constraints.**
  Interim goal (regardless of the method):
  Re-sum all quantum-gravity fluctuations (non-perturbative!)

Effective Actions in Quantum Gravity

- **Goal: predictions, contrast different theories, put constraints.**
  Interim goal (regardless of the method):
  Re-sum all quantum-gravity fluctuations (non-perturbative!)

- **Effective action from path integral over metrics**
  [= sum over all 1PI vacuum diagrams]

$$\Gamma_{\rm eff} \quad \Leftrightarrow \quad \int \mathcal{D}g_{\mu\nu} e^{iS[g_{\mu\nu}]}$$

Effective Actions in Quantum Gravity

- **Goal: predictions, contrast different theories, put constraints.**
  Interim goal (regardless of the method):
  Re-sum all quantum-gravity fluctuations (non-perturbative!)

- **Effective action from path integral over metrics**
  [= sum over all 1PI vacuum diagrams]

$$\Gamma_{\text{eff}} \quad \Leftrightarrow \quad \int \mathcal{D}g_{\mu\nu}\, e^{iS[g_{\mu\nu}]} \longrightarrow$$

Note:

- Can use different fundamental dof & symmetries;
- Expected to have/recover description in terms of effective actions (AS gravity, CDT, non-local gravity, string theory, etc).
- In discrete theories more difficult but same concepts applicable

Effective Actions in Quantum Gravity

- **Goal: predictions, contrast different theories, put constraints**
  Interim goal (regardless of the method):
  Re-sum all quantum-gravity fluctuations (non-perturbative!)

- **Effective action from path integral over metrics**
  [= sum over all 1PI vacuum diagrams]

$$\Gamma_{\text{eff}} \quad \Leftrightarrow \quad \int \mathcal{D}g_{\mu\nu} e^{iS[g_{\mu\nu}]}$$

- **Predictions:**

$$\frac{\delta\Gamma_{\text{eff}}}{\delta g_{\mu\nu}} = 0 \quad \Rightarrow$$ fully-quantum dynamics, fully-quantum solutions

Higher-order variations & contractions ⇒ dressed scattering amplitudes

Effective Actions in Quantum Gravity

Parametrize effective action in terms of form factors:

$$\Gamma_{\text{eff}} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}_{\text{HD}}\right)$$

$$\mathcal{L}_{\text{HD}} = \frac{1}{16\pi G}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3)\right)$$

Effective Actions in Quantum Gravity

Parametrize effective action in terms of form factors:

$$\Gamma_{\text{eff}} = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G} + \mathcal{L}_{\text{HD}}\right)$$

$$\mathcal{L}_{\text{HD}} = \frac{1}{16\pi G}\left(R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\rho\sigma}\mathcal{F}_3(\Box)R^{\mu\nu\rho\sigma} + \mathcal{O}(R^3)\right)$$

Different QG theories will yield different form factors.
Two-way approach:

- **Top-down**:
  - compute form factors from fundamental theories (path integral, renormalization group, etc.)
- **Bottom-up**:
  - Model building;
  - Constrain form factors using consistency constraints (positivity bounds, stability, black hole considerations, effective field theory arguments, etc.) and observations;

Non-localities in the QG effective actions

Different QG theories will yield different form factors. Examples:

Non-local gravity [Buoninfante, Koshlev, Mazumdar, Modesto, Tokareva, …]: $\mathcal{F}_i \propto \frac{e^{H(-\Box)}-1}{\Box}$

Spin foams [Borissova, Dittrich]: $\mathcal{F}_i \propto \frac{1}{a+b\Box}$

Asymptotic-safety models [Draper, Knorr, Ripken, Saueressig]: $\mathcal{F}_i \propto \tanh(a\Box)$

Causal dynamical triangulations [Knorr, Saueressig]: $\mathcal{F}_i \propto \Box^{-2}$

Polyakov-type operators in some string models [Polyakov]: $\mathcal{F}_i \propto \Box^{-1}$

$$\mathcal{F}_i(\Box) = \sum_{n=-\infty}^{\infty} c_n \Box^n$$

Non-localities in the QG effective actions

Different QG theories will yield different form factors. Examples:

Non-local gravity [Buoninfante, Koshlev, Mazumdar, Modesto, Tokareva, …]: $\mathcal{F}_i \propto \frac{e^{H(-\Box)} - 1}{\Box}$

Spin foams [Borissova, Dittrich]: $\mathcal{F}_i \propto \frac{1}{a + b\Box}$

Asymptotic-safety models [Draper, Knorr, Ripken, Saueressig]: $\mathcal{F}_i \propto \tanh(a\Box)$

Causal dynamical triangulations [Knorr, Saueressig]: $\mathcal{F}_i \propto \Box^{-2}$

Polyakov-type operators in some string models [Polyakov]: $\mathcal{F}_i \propto \Box^{-1}$

$$\mathcal{F}_i(\Box) = \sum_{n=-\infty}^{\infty} c_n \Box^n$$

Pheno models: <u>focus on IR non-localities</u>

- **IR deviation from GR and dynamical dark energy, seemingly compatible with observations**
[Amendola, Akrami, et al; Capozziello e al; Deser, Woodard; Maggiore et al; Wetterich; …]
- **Make popular regular black-hole models compatible with a principle of least action**
[Knorr, Platania]

Black hole entropy from IR non-localities

- **IR non-localities ⇒ large-scale deviations from GR (controlled for not too non-local operators?)**

- **Black hole entropy depends on higher-derivative/curvature corrections via Wald formula**

$$S_W = -2\pi \int_\Sigma \left(\frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \right) \epsilon_{\mu\nu}\epsilon_{\rho\sigma} dV_2^2$$

- **One-loop corrections** [Xavier, Kuipers] **⇒ Log corrections to area law**

- **General form factor within the annulus of convergence:**

$$\mathcal{F}_i(\Box) = \sum_{n=-\infty}^{\infty} c_n \Box^n$$

- **Positive-degree operators ⇒ sub-leading corrections to Bekenstein-Hawking area law** [Mazumdar et al; Myung]**:**

$$S_W = \frac{\mathcal{A}}{4G}(1 + \sum_{n>0} a_n\, r_H^{-2n})$$

- **Negative-degree terms, aka, IR non-localities?**
  - **Expectation**: dominant contribution over Bekenstein-Hawking term; how dominant?
  - **Computational caveat**: localization procedure, recursion formulas, large black holes

Black hole entropy from IR non-localities

Computational setup: large, static, spherically-symmetric asymptotically flat black holes.

Modified field equations will admit the asymptotic solutions [Knorr, Platania, '22]:

$$g_{\mu\nu} = \mathrm{diag}\left(-f_{tt}(r), f_{rr}^{-1}(r), r^2, r^2 \sin\theta\right) \qquad f_{tt}(r) \simeq 1 - \frac{r_s}{r} + \frac{A}{r^\alpha}, \quad f_{rr}(r) \simeq 1 - \frac{r_s}{r} + \frac{B}{r^\beta}$$

Focus on the **principle part of the form factors**:

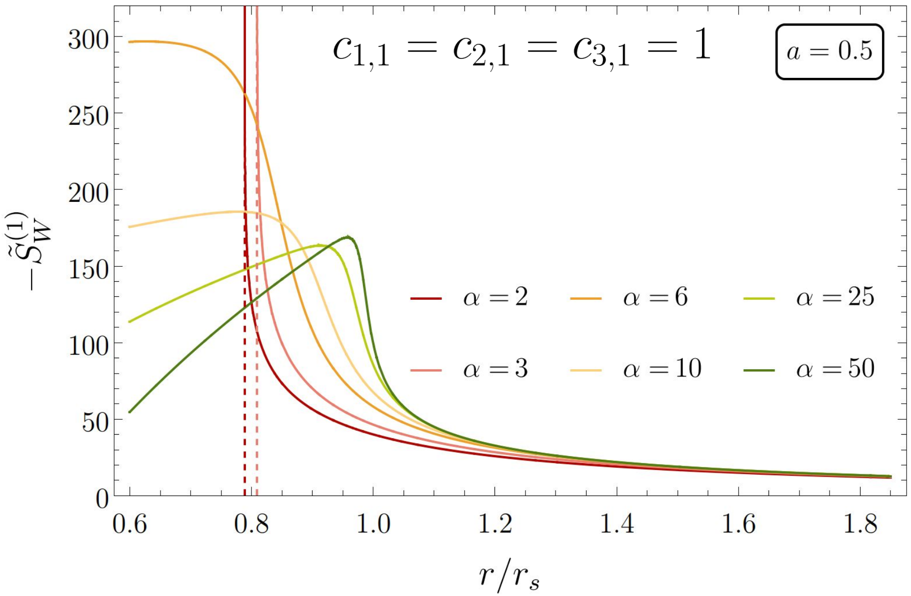
$$\mathcal{F}_i(\Box) = \sum_{n=1}^{N} F_{i,n} \, , \qquad F_{i,n} \equiv c_{i,n} \Box^{-n}$$

Intermediate step: localization procedure (the Wald formula cannot be directly applied otherwise)
⇒ Corresponding **dimensionless entropy contributions (after)**:

$$\tilde{S}_W^{(n)} \equiv \lim_{r \to r_H} \left(2 c_{1,n} \frac{1}{\Box^n} R_1 + c_{2,n} \frac{1}{\Box^n} R_2 - 4 c_{3,n} \frac{g}{f} \frac{1}{\Box^n} R_3 \right)$$

To be computed **recursively**. For n=1 [Knorr, Platania, '22]:

$$\Box^{-1} \phi(r) = \int_r^{R_x} \int_x^{R_y} dx\, dy \frac{-y^2 \phi(y)}{x^2 \sqrt{f_{rr} f_{tt}(x)}} \sqrt{\frac{f_{tt}(y)}{f_{rr}(y)}}$$



Lapse function, simple case:

$$f_{tt} = f_{rr} \simeq 1 - \frac{r_s}{r} + \frac{A}{r^\alpha}$$

The event horizon, when it exists, is located at:

$$r_H \simeq \left(1 + \frac{a}{(a-1)\,\alpha - 1}\right) r_s \quad a \equiv (\alpha+1) r_s^{-\alpha} A$$

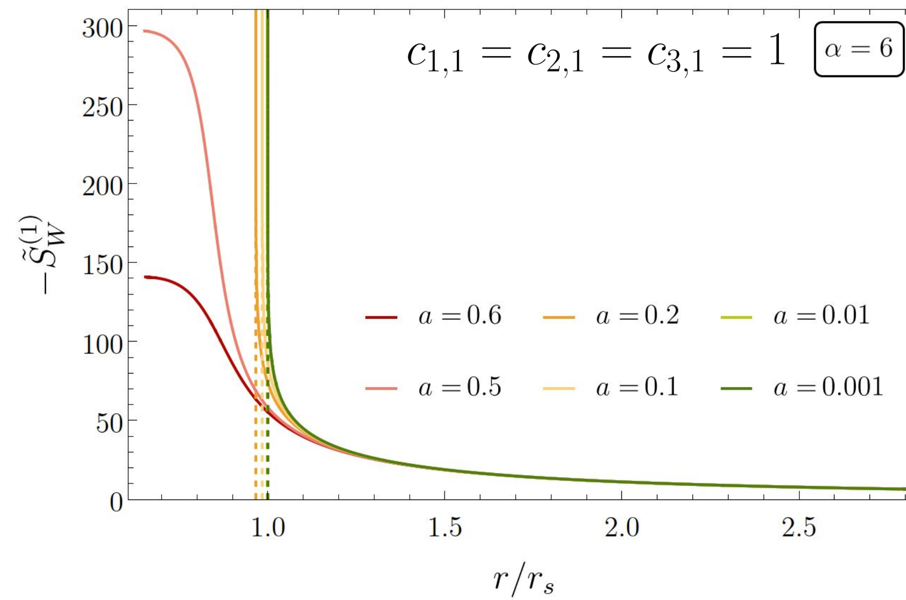
The first contribution to the entropy diverges anytime an event horizon exists.

<u>Note</u>:

- Divergence of the first contribution ⇒ divergence of all of the others

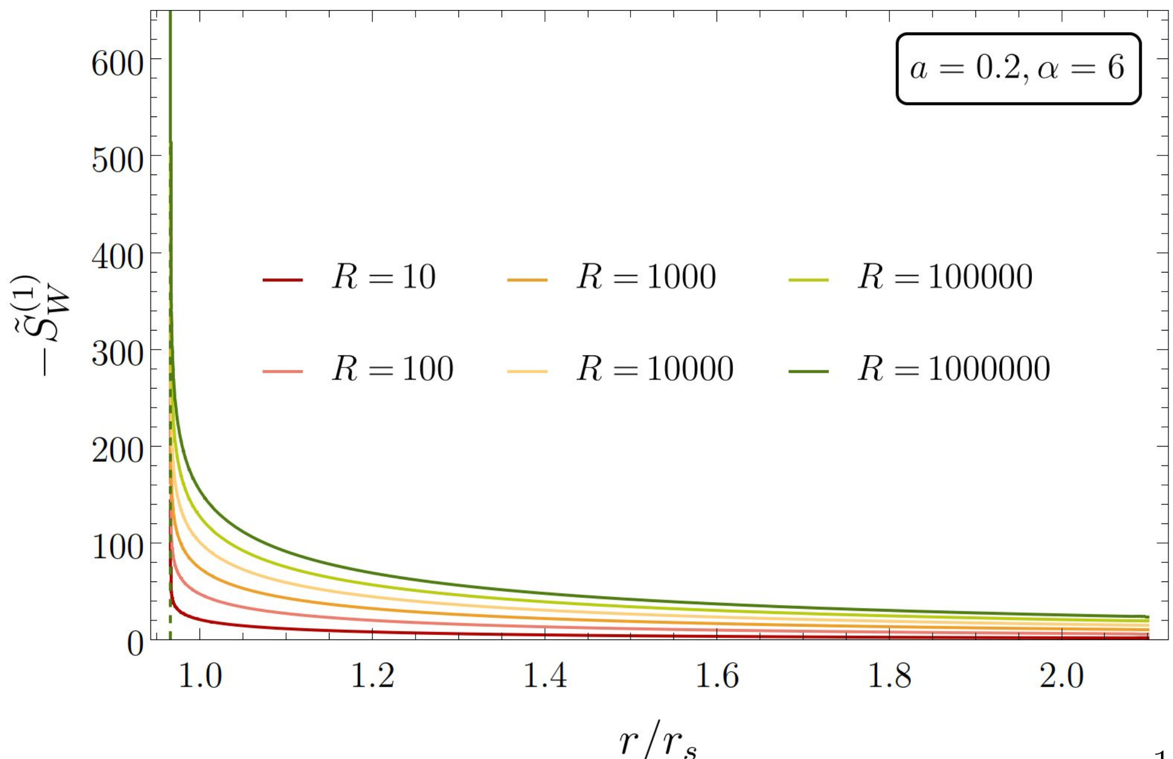
<u>The result is independent of</u>:

- The parameters one puts in
- Whether the lapse functions coincide
- Initial conditions



Independence of initial conditions

$$\Box^{-1}\phi(r) = \int_r^{R_x}\int_x^{R_y} dx\, dy \frac{-y^2\phi(y)}{x^2\sqrt{f_{rr}f_{tt}(x)}}\sqrt{\frac{f_{tt}(y)}{f_{rr}(y)}} \qquad R = R_x = R_y$$



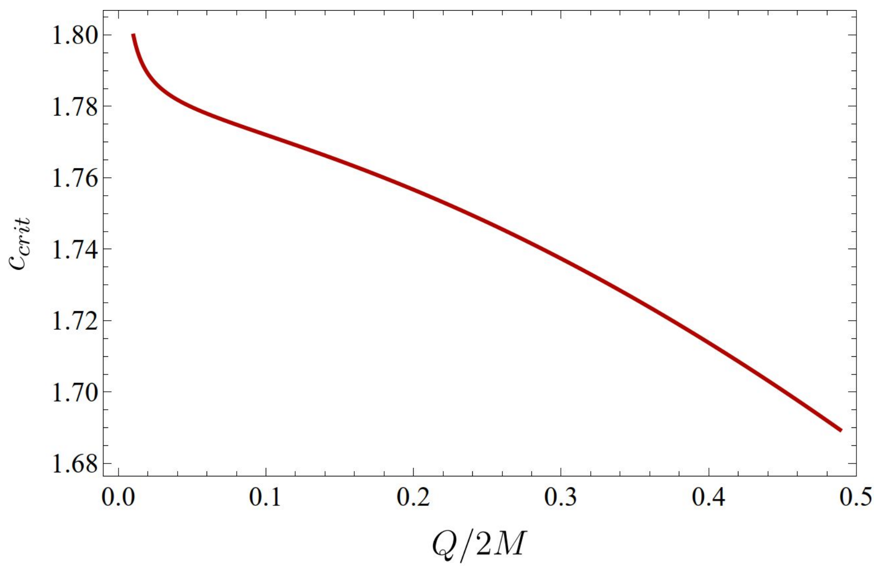
$c_{1,1} = c_{2,1} = c_{3,1} = 1$

Possible cancellations of divergences is non-trivial.

Case of charged black holes (analytical formulas computed)

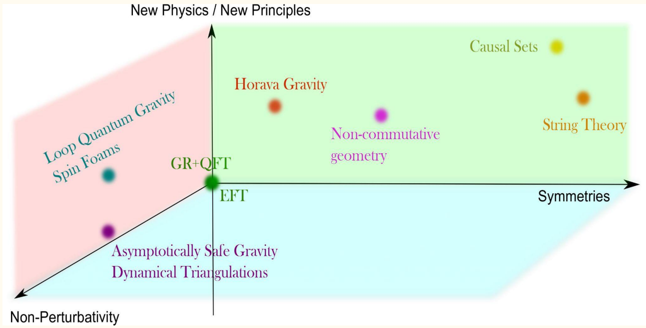
$$\tilde{S}_W^{(1)} = \lim_{r\to r_H} \left(c_{2,1}\Phi[R_2, r, R_x, R_y] - 4c_{3,1}\Phi[R_3, r, R_x, R_y]\right)$$

$$\tilde{S}_W^{(1)} = \begin{cases} \mathrm{sign}(c_{2,1}) \times \infty\,, & c_{2,1} > c_{\rm crit}(Q/2M)\, c_{3,1}\,, \\ -\mathrm{sign}(c_{2,1}) \times \infty\,, & c_{2,1} < c_{\rm crit}(Q/2M)\, c_{3,1}\,, \end{cases}$$



Summary and Conclusions

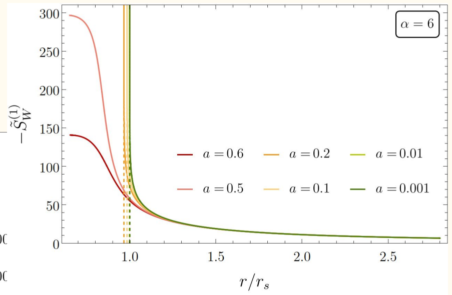
- Different approaches to quantum gravity

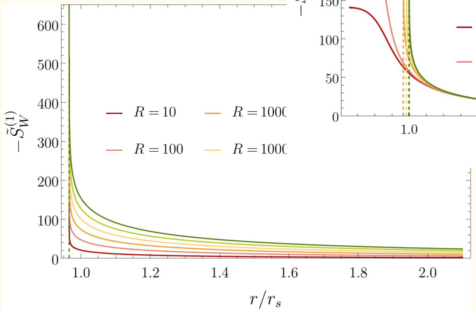


- Effective actions from different approaches
  ⇒ different effective non-localities

- IR non-localities in effective actions:
  - Phenomenologically appealing
  - Occur in various QG models

- Modified black hole entropy?
  Result: generally divergent for large black holes, modulo fine tuning
  ⇒ non-trivial test for pheno and QG models





Thank you!

Summary and Conclusions

- **Different approaches to quantum gravity**
- **Effective actions from different approaches**
  ⇒ different effective non-localities
- **IR non-localities in effective actions:**
  - **Phenomenologically appealing**
  - **Occur in various QG models**
- **Modified black hole entropy?**
  Result: generally divergent for large black holes, modulo fine tuning
  ⇒ non-trivial test for pheno and QG models