D-instanton Amplitudes in String Theory

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Bengaluru, April 2023

Collaborators

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Parallel developments

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+ Nathan Agmon, Minjae Cho

D-instantons

D-instantons are D-branes with Dirichlet boundary condition along all non-compact directions, including Euclidean time

– describe finite action (C/g_s) classical solutions in string theory

– analogous to instantons in quantum field theory

– give non-perturbative corrections to string amplitudes

$$e^{-C/g_s} \times \text{power series in } g_s$$

World-sheet theory of closed and open strings provide (formal) expressions for the D-instanton contribution to the ampltudes

Integrals over moduli spaces of Riemann surfaces with boundaries generate the series expansion in g_s multiplying e^{-C/g_s}

In this talk we shall focus on single D-instanton amplitudes for simplicity.

Systematics of D-instanton induced amplitudes

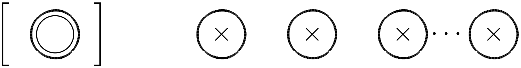
Individual world-sheets with boundaries on the D-instanton do not conserve energy / momentum

– disconnected world-sheets contribute even for generic values of external energy / momentum

For getting leading contribution to the D-instanton amplitude, we

– maximize the number of disks since each disk gives $1/g_s$

– can use as many annuli as we want since annuli $\sim (g_s)^0$

$\exp[-C/g_s]\ \ \exp$ 

$\times$: closed string vertex operator

At the next order there are more possibilities

$\mathbf{exp}[-\mathbf{C}/\mathbf{g_s}]$ **exp**

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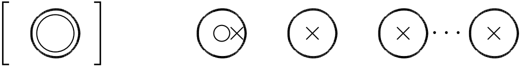


etc.

This way we can write down the expression for D-instanton induced amplitude to any order in the string coupling $\mathbf{g_s}$

However, the moduli space integrals diverge from regions of the moduli space where the Riemann surface degenerates

Example from c=1 bosonic string theory: **Balthazar, Rodriguez, Yin**

$$\text{Annulus partition function} = \int_0^\infty \frac{dt}{2t}\left(e^t - 1\right)$$

$$\text{Disk two point function} \propto \text{finite} + \frac{1}{2}\int_0^1 dy\, y^{-2}(1 + 2\omega_1\omega_2 y)$$

Annulus 1-point function

$$\propto \quad \text{finite} + \int_0^1 dv \int_0^{1/4} dx \left\{\frac{v^{-2} - v^{-1}}{\sin^2(2\pi x)} + 2\,\omega^2\, v^{-1}\right\}$$

For D-instantons in type IIB string theory in ten dimensions

$$\text{Annulus partition function} = \int_0^\infty \frac{dt}{2t}\,(8-8)$$

8 from NS sector, -8 from R sector

Naively the answer vanishes

However this gives results for instanton correction that are inconsistent with the prediction of S-duality

$\Rightarrow$ breakdown of AdS/CFT correspondence since the dual N=4 super-Yang-Mills is S-duality invariant.

In order to make sense of these divergences and extract a finite result we need ‘string field theory’

String field theory (SFT)

String field theory is a regular quantum field theory with infinite number of fields, one for each mode of the string

Perturbative amplitudes are given by sum of Feynman diagrams

Propagator $\propto (L_0)^{-1}$, L_0: World-sheet scaling generator

L_0 eigenvalues $\propto k^2 + m^2$, m: mass of the string mode

SFT is designed so that formally the sum of Feynman diagrams reproduce the world-sheet expression after using

$$(L_0)^{-1} = \int_0^\infty dt\, e^{-L_0 t}, \qquad t : \text{Schwinger parameter}$$

t's become the moduli of Riemann surfaces after change of variables

$$\text{SFT} \quad \Rightarrow \quad (L_0)^{-1} = \int_0^\infty dt\, e^{-L_0 t} \quad \Leftarrow \quad \text{world-sheet}$$

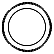
1. This is an identity for $L_0 > 0$

2. For $L_0 < 0$ the rhs diverges from $t \to \infty$ end but the lhs is finite and we can use lhs as the correct expression

3. For $L_0 = 0$ both sides diverge

However, on the lhs we sit on the pole of a propagator and insights from QFT can be used to make sense of this.

This is the essence of why string field theory is useful for dealing with divergences in the integrals over the moduli spaces of Riemann surfaces

$\exp\Big[$  $\Big] = \exp\Big[-\int_0^\infty \frac{dt}{2t}\, Z(t)\Big]$

t $\propto$ ratio of circumference to the width of the cylinder / annulus

$$Z(t) = \mathrm{Tr}\left\{(-1)^F e^{-t L_0} b_0 c_0\right\}$$

Tr is trace over open string states on the D-instanton

$b_0 c_0$ is needed to remove ghost zero modes

$$Z(t) = \sum_b e^{-t\, h_b} - \sum_f e^{-t\, h_f}$$

h_b, h_f: L_0 eigenvalues of bosonic / fermionic open string states that are annihilated by b_0 (Siegel gauge)

If h_b or $h_f \leq 0$, then the integral diverges from large t region.

Strategy for dealing with large t divergence:

1. Use the identities, valid for $h_b, h_f > 0$,

$$\exp\left[\int \frac{dt}{2t}\left(e^{-th_b} - e^{-th_f}\right)\right] = \sqrt{\frac{h_f}{h_b}}$$

$$h_b^{-1/2} = \int \frac{d\psi_b}{\sqrt{2\pi}} e^{-\frac{1}{2}h_b\psi_b^2}, \qquad \psi_b : \text{grassmann even}$$

$$h_f = \int du_f\, dv_f\, e^{-h_f u_f v_f}, \qquad u_f, v_f : \text{grassmann odd}$$

2. Interpret the modes ψ_b, u_f, v_f as open string fields (D=0) and the exponent as open string field theory action in Siegel gauge

3. Modes with $h_b < 0$ are tachyonic modes and integration over them can be carried out along the steepest descent contour

4. Modes with $h_b = 0$ and $h_f = 0$ represent respectively the bosonic and fermionic zero modes

– need to be treated carefully.

Origin of zero modes

1. Bosonic zero modes can arise from the freedom of translating the instanton along flat directions e.g. Euclidean time

Remedy: Change variables from bosonic zero modes to D-instanton position y, picking up the Jacobian factor.

Integration over y has to be done at the end and produces the energy momentum conserving delta function

Similar treatment is needed for the fermion zero modes associated with broken supersymmetry.

2. We often have fermion zero modes coming from ghost sector

$$c_1 c_{-1}|0\rangle, \qquad |0\rangle$$

They are results of wrongly fixing the U(1) 'gauge symmetry' on the instanton

Consider the gauge invariant open string field theory on a Dp-brane

– **has a U(1) gauge field.**

Action:

$$\int d^{p+1}x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \left(\frac{1}{\sqrt{2}}\, \partial^\mu A_\mu - \phi \right)^2 \right]$$

ϕ: **mode associated with the state** $c_0 e^{ik.X}(0)|0\rangle$

– **not present in the Siegel gauge but is present in the gauge invariant theory**

Gauge transformation:

$$\delta A_\mu = \sqrt{2}\, \partial_\mu \theta(x), \quad \delta\phi = \Box\theta(x)$$

$$\mathbf{S} = \int \mathbf{d^{p+1}x} \left[-\frac{1}{4}\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu} - \left(\frac{1}{\sqrt{2}}\,\partial^\mu \mathbf{A}_\mu - \phi \right)^2 \right]$$

$$\delta \mathbf{A}_\mu = \sqrt{2}\,\partial_\mu \theta(\mathbf{x}), \quad \delta\phi = \Box\theta(\mathbf{x})$$

Siegel gauge $\phi = 0$ leads to gauge fixed action including ghosts:

$$\int \mathbf{d^{p+1}x} \left[-\frac{1}{2}\mathbf{A}^\mu \Box \mathbf{A}_\mu - \mathbf{u}\Box\mathbf{v} \right], \qquad \mathbf{u}, \mathbf{v} : \ \textbf{ghosts}$$

On D-instanton, there is no A_μ and all fields are x independent

$\Rightarrow \mathbf{u}\Box\mathbf{v} = \mathbf{0}$

$\Rightarrow$ **leads to ghost zero modes**

– arise since we are attempting to gauge fix a rigid symmetry with parameter θ under which $\delta\phi = 0$

Remedy: Undo the gauge fixing by using a gauge invariant form of the path integral

1. Integrate over ϕ and drop the integration over the ghosts

$$\int \mathbf{d}\phi\, \mathbf{e}^{-\phi^2} = \sqrt{\pi}$$

2. Divide by the volume of the gauge group, given by the period of θ

– can be found by carefully comparing the string field theory gauge transformation laws with $\psi \to \mathrm{e}^{\mathrm{i}\alpha}\psi$ where α has period 2π.

ψ: any state of the open string with one end on the instanton

Example: c=1 bosonic string theory

World-sheet theory has

1. A scalar X describing time direction

2. A Liouville field χ_L with central charge 25

– describes space direction with a potential

3. b,c ghost system with central charge -26

This theory has ZZ instanton with Dirichlet boundary condition on X and χ_L and action $1/g_s$

Zamolodchikov, Zamolodchikov

Exponential of the annulus diagram is:

$$\exp\left[\int_0^\infty \frac{dt}{2t}\, Z(t)\right] = \exp\left[\int_0^\infty \frac{dt}{2t}\, (e^t - 1)\right]$$

e^t is from a tachyon with $h_b = -1$

In the path integral representation this contributes $\sqrt{1/h_b} = i$

-1 is the result of

1. A bosonic zero mode associated with translation along X

2. Two fermionic zero modes from the ghost

Contribution from translational zero mode:

$$\int \frac{d\psi_b}{\sqrt{2\pi}} = \frac{1}{\sqrt{2\pi}} \int \frac{dy}{\pi\, g_o \sqrt{2}}$$

$g_o = (g_s/2\pi^2)^{1/2}$ is the 'open string coupling'

y: D-instanton location along time direction, couples via $e^{iyE_{total}}$

$\int$ dy produces $2\pi\delta(E_{total})$ at the end

Ghost zero mode integral is replaced by

$$\int d\phi\, e^{-\phi^2} \Big/ \int d\theta = \frac{\sqrt{\pi}}{2\pi/g_o}$$

Net normalization $i/(4\pi^2)$ agrees with a dual matrix model result.

Divergences arise also at higher order calculation.

The disk two point function has divergences from the region where one vertex operator approaches the boundary.

Annulus one point function has divergences from the region where the vertex operator approaches the boundary and / or the circumference becomes large.

e.g. in c=1 bosonic string theory

$$\text{Disk two point function} \propto f_{\text{finite}} + \frac{1}{2}\int_0^1 dy\, y^{-2}(1 + 2\omega_1\omega_2 y)$$

Annulus 1-point function

$$\propto \quad g_{\text{finite}} + \int_0^1 dv \int_0^{1/4} dx \left\{ \frac{v^{-2} - v^{-1}}{\sin^2(2\pi x)} + 2\,\omega^2\, v^{-1} \right\}$$

Strategy:

1. Express the amplitudes as sum over SFT Feynman diagrams

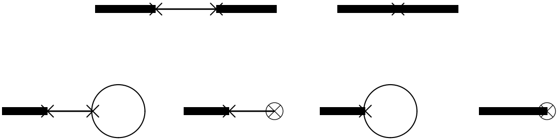
– automatically replaces the tachyon contribution by $1/h$ where h is the L_0 eigenvalue

2. Remove the zero mode contribution to the propagators since they are to be integrated at the end or removed altogether.

3. Add the propagator of the field ϕ that was not present in the world-sheet formulation but should be present.

4. Account for corrections to the jacobian factors

Feynman diagrams contributing to disk two point function and annulus one point function



Thin line: Open string   Thick line: closed string

$\times$: disk interaction vertex $\otimes$: annulus interaction vertex

1. Remove zero mode contribution from each open string propagator

2. Put back explicit ϕ contribution in each open string propagator.

Some results in c=1 bosonic string theory:

World-sheet gives

$$\text{Disk two point function} \propto f_{\text{finite}} + \frac{1}{2}\int_0^1 dy\, y^{-2}(1+2\omega_1\omega_2 y)$$

Annulus 1-point function

$$\propto \quad g_{\text{finite}} + \int_0^1 dv \int_0^{1/4} dx \left\{ \frac{v^{-2}-v^{-1}}{\sin^2(2\pi x)} + 2\,\omega^2\, v^{-1} \right\}$$

String field theory gives

$$\text{Disk two point function} \propto f_{\text{finite}} - \frac{1}{2}(1 - 2\,\omega_1\,\omega_2 \ln \lambda^2)$$

$$\text{Annulus 1-point function} \propto g_{\text{finite}} + \frac{1}{2}\,\omega^2 \ln \frac{\lambda^2}{4}$$

λ: an arbitrary constant parameter labelling string field theory

String field theory gives disk two point function and annulus one point function to be proportional to

$$\mathbf{f}(\omega_1, \omega_2) = \mathbf{f}_{\text{finite}} - \frac{1}{2}(1 - 2\,\omega_1\,\omega_2 \ln \lambda^2)$$

$$\mathbf{g}(\omega) = \mathbf{g}_{\text{finite}} + \frac{1}{2}\,\omega^2 \ln \frac{\lambda^2}{4}$$

Final result for an amplitude involving n external closed strings of energy $\omega_1, \cdots, \omega_n$ is proportional to

$$\sum_{i<j} \mathbf{f}(\omega_i, \omega_j) + \sum_i \mathbf{g}(\omega_i)$$

This is independent of λ and agrees with the results from the dual matrix model after numerical evaluation of the finite parts

Balthazar, Rogriguez, Yin

When the result is known from a dual description, this procedure produces the correct result in all cases that have been studied.

1. **c=1 bosonic string theory**

   Balthazar, Rodriguez, Yin; A.S; Eniceicu, Mahajan, Murdia, A.S.

2. **c<1 bosonic string theory** — Eniceicu, Mahajan, Murdia, A.S.

3. **Type IIB in D=10** — A.S.; Agmon, Balthazar, Cho, Rodriguez, Yin

4. **Type IIA / IIB on CY_3** — Alexandrov, A.S., Stefanski

5. **$\hat{c} = 1$ type 0B string theory** — Balthazar, Rodriguez, Yin; Chakravarty, A.S.

6. **IIA/IIB on CY_3 orientifolds** — Alexandrov, Firat, Kim, A.S., Stefanski

7. **Sine-Liouville deformation of c=1 bosonic string theory**

   Alexandrov, Mahajan, A.S., work in progress

Unitarity

Based on our understanding of D-instanton amplitudes, one can also analyze unitarity of these amplitudes

Result: The only source of unitarity violation is in the imaginary part of the exponential of the annulus partition function

– related to the tachyonic modes on the instanton

Conclusion

World-sheet theory, aided by string field theory, provides a fully systematic procedure for computing D-instanton contribution to an amplitude

Besides being of practical use, this can be used to gain deeper understanding of string theory, e.g,

– testing duality conjectures

– role of resurgence

etc.