Quantum black holes as classical space factories

Luca Smaldone
in collaboration with A. Iorio

Institute of Theoretical Physics,
University of Warsaw

Quantum Gravity and Cosmology 2023, 20 April 2023

Contents

Preliminaries: Xons

Bekenstein Bound and Xons

Bekenstein's argument that a black hole (BH) reaches the maximal entropy at disposal of a physical system[1] led to two main proposals:

- The degrees of freedom (dof) responsible for the BH entropy have to take into account both matter and spacetime and hence must be of a new, more fundamental nature than the dof we know, here we call such dof Xons[2]
- The Hilbert space $\mathcal{H}$ of the Xons of a given BH is necessarily finite dimensional[3]

[1] J. D. Bekenstein, Phys. Rev. D **23**, 287 (1981).
[2] G. Acquaviva, A. Iorio and M. Scholtz, Ann. Phys. **387**, 317 (2017).
[3] N. Bao, S. M. Carroll and A. Singh, Int. J. Mod. Phys. D **26**, no. 12, 1743013 (2017).

Fermionic Xon model of BH evaporation

- We assume our system has only a finite number N of quantum levels (slots) to be filled (local quantum system)
- We assume that Xons are fermions $\Rightarrow$ Each quantum level can be filled by no more than one fermion $\Rightarrow$ $\dim\mathcal{H} < \infty$.
- Before evaporation, BH state is described by free Xons, which fill all slots.
- Evaporation consists of steady process:
  $N \to (N-1) \to (N-2) \to \cdots$. That is, the number of *free* Xons steadily decreases
- During evaporation Xons rearrange into quasi-particles and the spacetime they live in $\Rightarrow$ Intrinsic notion of interior (BH) and exterior (environment)[4].

[4] G. Acquaviva, A. Iorio and L.S., Phys. Rev. D **102**, 106002 (2020).

BH state in Xon model

BH (b)/environment (a) state

$$|\Psi(\sigma)\rangle \;=\; \prod_{i=1}^{N} \sum_{n_i=0,1} C_i(\sigma) \left(a_i^\dagger\right)^{n_i} \left(b_i^\dagger\right)^{1-n_i} |0\rangle_{\mathrm{I}} \otimes |0\rangle_{\mathrm{II}}$$

with

$$C_i = (\sin\sigma)^{n_i} (\cos\sigma)^{1-n_i}$$

σ is an interpolating parameter, which describes the evolution of the system, from $\sigma = 0$ till $\sigma = \pi/2$. $\mathcal{H}$ has dimension

$$\Sigma \;=\; 2^N \;=\; e^{\mathcal{S}_{max}}$$

Elastic theory vs from QFT

- The division of a and b modes is artificial: No explanation of the mechanism behind environment (space and matter) formation!
- A possible solution comes from condensed matter physics: known relation between classical theory of defects and a Einstein–Cartan description of gravity[5]
- Analog models of gravity in Dirac materials[6]
- By using the boson method, classical theory of defects and elasticity theory can be derived by an underlying QFT[7]

[5] M. O. Katanaev and I. V. Volovich, Annals Phys. **216**, 1 (1992)
[6] G. Acquaviva, A. Iorio, P. Pais, and L. Smaldone, Universe **8** (9), 455 (2022)
[7] M. Wadati, H. Matsumoto, and H. Umezawa, Phys. Rev. B **18**, 4077 (1978)

From Xons to the emergent world: plane

We can apply the same idea to our *quasiparticle picture*[8]:

$$\text{Quasiparticle picture} \begin{cases} \text{Elasticity theory} - \text{Classical geometry} \\ \qquad\qquad\Uparrow \quad (\text{Crystal Gravity}) \\ \text{Quantum field theory of crystal} - \text{defects} \\ \qquad\qquad\Uparrow \quad (\text{Boson method}) \\ \text{Fundamental quantum dynamics} \quad (\text{QED}\ ,\ X\text{ons}) \end{cases}$$

[8] A. Iorio and L. Smaldone, [arXiv:2302.04847 [hep-th]].

The boson method

Dynamical map

Field equation

$$\Lambda(\partial)\,\Psi(x) \;=\; j[\Psi](x)$$

$\Lambda(\partial)$ is a differential operator. Yang–Feldman equation[9]:

$$\Psi(x) \;=\; \Psi_0(x) \;+\; \left(\Lambda^{-1} \star j[\Psi]\right)(x)\,, \qquad \Lambda(\partial)\Psi_0(x) \;=\; 0\,,$$

$\Lambda(\partial)\Psi_0 = 0$. Iteratively solved by the dynamical map[10]:

$$\Psi(x) \;=\; \Psi[x; \Psi_0(x)]$$

[9]C.-N. Yang and D. Feldman, Phys. Rev. **79**, 972 (1950).
[10]H. Umezawa, Advanced field theory: Micro, macro, and thermal physics (AIP, 1993).

Example: Real scalar field

Example: Consider $\mathcal{L} = \frac{1}{2}(\partial\Psi)^2 - \frac{m^2}{2}\Psi^2 - V(\Psi)$. Then

$$\Lambda(\partial) \ = \ \Box + m^2 \,, \qquad j[\Psi] = \frac{\partial V}{\partial \Psi}$$

Yang–Feldman equation ($\Psi_0 = \Psi_{in}$)

$$\Psi(x) \ = \ \sqrt{Z}\,\Psi_{in}(x) \ + \ \int \mathrm{d}^4 y\, \Delta_R(x-y)\, j(x)$$

Δ_R is the retarded propagator. Dynamical map[11]

$$\Psi(x) = v + \sum_{n=1}^{\infty} \int \mathrm{d}^4 x_1 \ \ldots \ \mathrm{d}^4 x_n \, F_n(x; x_1, \ldots, x_n) \ : \Psi_{in}(x_1) \ \ldots \ \Psi_{in}(x_n) :$$

$$F_n(x; x_1, \ldots, x_n) \ = \ Z^{\frac{n+1}{2}}\, \Lambda_{x_1} \ldots \Lambda_{x_n} \, \langle R[\Psi(x)\, \Psi(x_1) \ \ldots \ \Psi(x_n)] \rangle_c$$

[11] S. S. Schweber, *An Introduction to Relativistic Quantum Field Theory*

Boson transformation theorem

Consider $\Psi_0[x; \psi(x), \varphi_1(x), \ldots, \varphi_N(x)]$. Then

$$\Psi(x) \;=\; \Psi[x; \psi(x), \varphi_1(x), \ldots, \varphi_N(x)]\,,$$

If φ_j are bosons, we can perform the boson transformation

$$\varphi_j(x) \;\to\; \varphi_j^f(x) \;\equiv\; \varphi_j(x) \;+\; f_j(x)\,,$$

f_j are c-number functions and φ_j^f satisfy the same equations as φ_j. Then[12]

$$\Lambda(\partial)\Psi^f(x) \;=\; j\left[\Psi^f\right](x)$$

with $\Psi^f(x) \equiv \Psi[x; \psi(x), \varphi_1^f(x), \ldots, \ldots, \varphi_N^f(x)]$

[12] H. Matsumoto, N. J. Papastamatiou, and H. Umezawa, Nucl. Phys. B **82**, 45 (1974)

Solutions with defects

When

$$[\partial_\mu \, , \partial_\nu] \, f_j(x) \;\neq\; 0 \, ,$$

Ψ^f describes a solution with topological defects[13].

System invariant under a Lie group G. Lie algebra generators:

$$[t_a \, , \; t_b] \;=\; iC_{abc} \, t^c$$

Spontaneous symmetry breaking (SSB)

$$Q_a|0\rangle \;\neq\; 0$$

The φ_js will be Nambu–Goldstone (NG) fields.

[13] L. Leplae and H. Umezawa, J. Math. Phys. **10**, 2038 (1969)

Classical gauge field

Boson transformed field[14]

$$\Psi^f(x) \;=\; U(x)\,\tilde{\Psi}(x)$$

$\tilde{\Psi}(x)$ is regular. $U[f](x)$ is a classical gauge transformation of G. Then

$$A_\mu(x) \;=\; i\,U^{-1}(x)\partial_\mu U(x)\,, \qquad A_\mu(x) \;=\; t_a\,A_\mu{}^a(x)$$

and

$$F_{\mu\nu}{}^a(x) \;=\; \partial_\mu A_\nu{}^a(x) - \partial_\nu A_\mu{}^a(x) + \; C_{abc}\,A_\mu{}^b(x)\,A_\nu{}^c(x)$$

are classical gauge fields depending on f.

[14] H. Matsumoto, H. Umezawa, and M. Umezawa, Fortsch. Phys. **29**, 441 (1981)

Example: The $\lambda\Psi^4$ model

$$\mathcal{L} = \partial_\mu \Psi^\dagger \partial^\mu \Psi - \mu^2 \Psi^\dagger \Psi - \frac{\lambda}{4} |\Psi^\dagger \Psi|^2$$

SSB when $\mu^2 < 0$ and $\langle \Psi \rangle = v = \sqrt{-2\mu^2/\lambda}$. Take $\Psi(x) \equiv (\rho(x) + v) e^{i\chi(x)}$. Equations of motion

$$\left[\Box - (\partial_\mu \chi)^2 + m^2\right] \rho + \frac{3}{2} m\, g\, \rho^2 + \frac{1}{2} g\, \rho^3 = v\, (\partial_\mu \chi)^2$$

$$\partial_\mu \left[(\rho + v)^2 \partial^\mu \chi\right] = 0\,.$$

$g = \sqrt{\lambda}$ and $m^2 = \lambda v^2 > 0$.

in-fields and dynamical map

Asymptotic fields choice the equations

$$\left[\Box - (\partial_\mu \chi_{in})^2 \, + \, m^2\right] (\rho_{in} \, + \, v) \, = \, 0$$

$$\partial_\mu \left[(\rho_{in} + v)^2 \, \partial^\mu \chi_{in}\right] \, = \, 0 \, ,$$

i.e. $\left(\Box \, + \, m^2\right) \Psi_{in} \, = \, 0, \; \Psi_0 = \Psi_{in} \equiv (\rho_{in} + v) \, e^{i\chi_{in}}.$ Dynamical map[15]

$$\Psi(x) \, = \, T_C \left\{ (\rho_{in}(x) + v) e^{i\chi_{in}(x)} \exp\left[-i \int_C d^4y \mathcal{L}^I_{in}(y)\right]\right\}$$

[15] M. Blasone, P. Jizba and G. Vitiello, *Quantum Field Theory and Its Macroscopic Manifestations*, (World Scientific, 2011).

The boson method and the vortex solution

C is a closed time-path[16], $\mathcal{L}_{in}^{I}$ is the interaction Lagrangian in Dirac picture. Boson transformation $\chi_{in}(x) \to \chi_{in}^{f}(x) = \chi_{in}(x) - f(x)$. Then

$$\psi^f(x) \;=\; e^{-if(x)}\, T_C \left\{ (\rho_{in}(x) + v) e^{i\chi_{in}(x)} \exp\left[-i \int_C d^4 y \mathcal{L}_{in}^{I}(y) \right] \right\}$$

Then

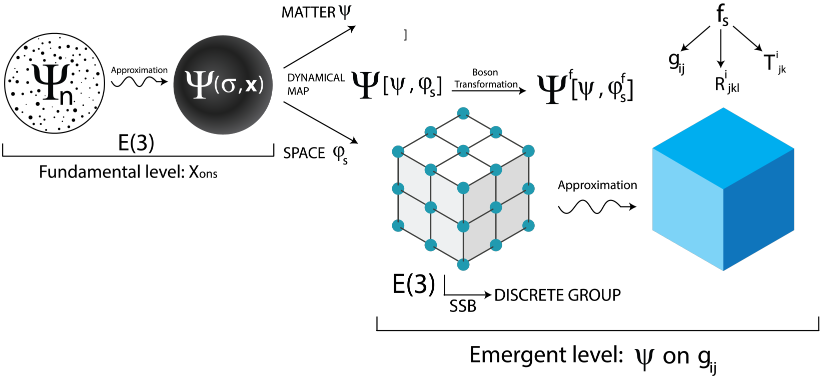
$$U(x) \;=\; e^{-if(x)}\,, \qquad A_\mu(x) \;=\; \partial_\mu f(x)\,, \qquad F_{\mu\nu}(x) \;=\; [\partial_\mu, \partial_\nu] f(x)$$

Vortex along z-axis when $f = \theta$ (azimuthal angle).

[16] J. Schwinger, J. Math. Phys. **2**, 407 (1961)

Emergence of space and matter

Emergence of space and matter



$E(3)$ invariant spin model

Spin doublet[17]

$$\Psi(x) \; = \; \begin{pmatrix} \Psi_\uparrow(x) \\ \Psi_\downarrow(x) \end{pmatrix}$$

$E(3)$ Lie algebra

$$[P_a \, , \, P_b] \; = \; 0$$

$$[P_a \, , \, J_b] \; = \; i\,\varepsilon_{abc} P_c$$

$$[J_a \, , \, J_b] \; = \; i\,\varepsilon_{abc}\, J_c$$

Consider SSB of such symmetry to a discrete group

[17] A. Iorio and L. Smaldone, [arXiv:2302.04847 [hep-th]].

Counting the NG bosons

Redundancy in NG counting if[18]

$$\int \mathrm{d}^3x \sum_a c_a(x)\, j_a^0(x)|0\rangle \;=\; 0$$

In our case

$$\int \mathrm{d}^3x\, j_a(x)|0\rangle \;=\; \int \mathrm{d}^3x\, \varepsilon_{abc} x^b p_c(x)|0\rangle + \int \mathrm{d}^3x\, \Sigma_a(x)|0\rangle$$

with

$$P_a = \int \mathrm{d}^3x p_a(x)\,, \qquad J_a = \int \mathrm{d}^3x j_a(x) \qquad S_a \;=\; \int \mathrm{d}^3x \Sigma_a(x)$$

and $J_a = L_a + S_a$

[18] H. Watanabe and H. Murayama, Phys. Rev. Lett. **110**, 181601 (2013)

Counting the NG bosons (2)

When $P_a|0\rangle \neq 0$, $S_a|0\rangle = 0$

$$\int d^3\, x\, j_a(x)|0\rangle \;=\; \int d^3 x\, \varepsilon_{abc} x^b p_c(x)|0\rangle$$

and we have 3 NG bosons $X^a(x)$. When $P_a|0\rangle \neq 0$, $S_a|0\rangle \neq 0$, we have 6 NG modes, X^a, Θ^a.

Table 1: Main features of the different SSB phases.

	$S\|0\rangle = 0$	$S\|0\rangle \neq 0$
NG fields	X^a	$X^a \;,\; \Theta^a$
${A_i}^a$	${e_i}^a \neq \delta_i^a \,, \quad {\omega_i}^a = 0$	${e_i}^a \neq \delta_i^a \,, \quad {\omega_i}^a \neq 0$
${F_{ij}}^a$	${T_{ij}}^a \neq 0 \,, \quad {R_{il}}^a = 0$	${T_{ij}}^a \neq 0 \,, \quad {R_{il}}^a \neq 0$

The case $S_a|0\rangle = 0$

When only $P_a|0\rangle \neq 0$:

$$\Psi(x) \ = \ \Psi_0(x) \, + \, \ldots \, , \qquad \Psi_0(x) \ = \ \Psi_0[x; \psi(x), X^a(x)]$$

ψ is a fermion field (matter-radial mode). Boson transformation

$$X^a(x) \ \to \ X^a_u(x) \ \equiv \ X^a(x) \, + \, u^a(\mathbf{x})$$

Gauge transformation

$$U(\mathbf{x}) \ = \ \exp\left(-i \, y^a(\mathbf{x}) \, \mathcal{P}_a\right) \, ,$$

$y^a(\mathbf{x})$ is a functional of u, $\mathcal{P}_a$ are (2×2 matrix) generators of translations. We use the indices i, j, k for x^i. y^a form a set of flat coordinates, while x^j are now curvilinear coordinates.

Triads, metric and torsion

Classical gauge field

$$A_j(\mathbf{x}) \;=\; e_j{}^a(\mathbf{x})\,\mathcal{P}_a\,, \qquad e_j{}^a(\mathbf{x}) \;\equiv\; \partial_j y^a(\mathbf{x})$$

$e_j{}^a$ are triads. Metric tensor

$$g_{ij}(\mathbf{x}) \;\equiv\; \delta_{ab}\,\partial_i y^a(\mathbf{x})\,\partial_j y^b(\mathbf{x}) \;=\; \delta_{ab}\,e_i{}^a(\mathbf{x})\,e_j{}^b(\mathbf{x}) \tag{1}$$

When $[\partial_i, \partial_j] y^a \neq 0$, non-trivial torsion

$$F_{ij}(\mathbf{x}) \;=\; T_{ij}{}^a(\mathbf{x})\,\mathcal{P}_a\,, \quad T_{ij}{}^a(\mathbf{x}) \;=\; \partial_i e_j{}^a(\mathbf{x}) - \partial_j e_i{}^a(\mathbf{x})$$

The case $S_a|0\rangle \neq 0$

Dynamical map of Xon field

$$\Psi(x) \;=\; \Psi[x;\psi(x),X^a(x),\Theta^a(x)]$$

Boson transformation

$$X^a(x) \;\;\to\;\; X^a_u(x) \;\equiv\; X^a(x) \,+\, u^a(\mathbf{x})$$

$$\Theta^a(x) \;\;\to\;\; \Theta^a_\theta(x) \;\equiv\; \Theta^a(x) \,+\, \theta^a(\mathbf{x})$$

and

$$\Psi^{u,\theta}(x) \;=\; \Psi^{u,\theta}[x;\psi(x),X^a_u(x),\Theta^a_\theta(x)]$$

Geometric tensors from boson transformation

Gauge matrix

$$U(\mathbf{x}) \;=\; \exp\left(-i\,\theta^a(\mathbf{x})\,\mathcal{J}_a - i\,y^a(\mathbf{x})\,\mathcal{P}_a\right)$$

$\mathcal{P}_a$ and $\mathcal{J}_a$ form a 2×2 matrix representation of the $e(3)$. Gauge field

$$A_j(\mathbf{x}) \;=\; \omega_j{}^a(\mathbf{x})\,\mathcal{J}_a \;+\; e_j{}^a(\mathbf{x})\,\mathcal{P}_a$$

$\omega_j{}^a \equiv \partial_j\theta^a$ is the spin connection, $e_j{}^a \equiv \partial_j y^a$ is the triad,
$F_{ij}(\mathbf{x}) \;=\; R_{ij}{}^a(\mathbf{x})\,\mathcal{J}_a \;+\; T_{ij}{}^a(\mathbf{x})\,\mathcal{P}_a\,$, where

$$\begin{aligned}
R^a_{ij}(\mathbf{x}) \;&=\; \partial_i\omega_j{}^a(\mathbf{x}) - \partial_j\omega_i{}^a(\mathbf{x}) \;+\; \varepsilon_{abc}\,\omega_i{}^b(\mathbf{x})\omega_j{}^c(\mathbf{x}) \\
T_{ij}{}^a(\mathbf{x}) \;&=\; \partial_i e_j{}^a(\mathbf{x}) - \partial_j e_i{}^a(\mathbf{x}) \;+\; \varepsilon_{abc}\,e_i{}^b(\mathbf{x})\,\omega_j{}^c(\mathbf{x})
\end{aligned}$$

Fermion field on curved space

For small u^a, Ψ_0 we use the ansatz

$$\Psi_0(x) \ = \ e^{-i[(x^a+X^a(x))\mathcal{P}_a+\Theta_a(x)\mathcal{J}_a]}\,\psi(x)\,,$$

Then $y^a(x) = x^a + u^a(\mathbf{x})$:

$$\Psi_0^{u,\theta}(x) \ = \ e^{-i[(y^a(\mathbf{x})+X^a(x))\mathcal{P}_a+(\Theta_a(x)+\theta^a(\mathbf{x}))\mathcal{J}_a]}\,\psi(x)\,.$$

When $\Lambda(\partial) = \gamma^\mu\partial_\mu$, γ^μ being 2×2 matrices, and on the X vacuum:

$$i\,\gamma^\mu D_\mu\,\psi(x) \ = \ 0\,, \qquad D_\mu \ = \ \left(\partial_\sigma, \partial_j + e_j{}^a(\mathbf{x})\,\mathcal{P}_a + \omega_j{}^a(\mathbf{x})\,\mathcal{J}_a\right)\,,$$

Conclusions

Conclusions

- BH evaporation can be described by Xons interaction
- In order to explain the emergence of space and matter, we use a continuous (field) approximation
- The boson method gives a convenient tool for studying defects in QFT
- We apply the boson method to a fermion theory with SSB of $E(3)$ symmetry: emergence of classical geometric tensors and QFT equation in curved space
- The method can also be applied to condensed matter systems, as analog gravity models[19]

[19] A. Iorio and L.S., in preparation

Thank you for the attention!