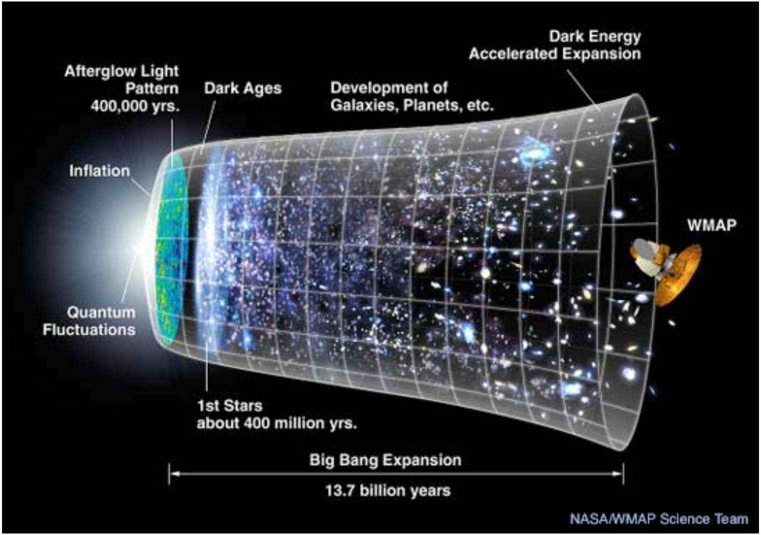
What preceded quasi-de Sitter (inflationary) stage in the early Universe?

Alexei A. Starobinsky

Landau Institute for Theoretical Physics RAS, Moscow - Chernogolovka, Russia

International virtual conference "Quantum Gravity and Cosmology"
21.04.2023


Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
400,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
WMAP
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.7 billion years
NASA/WMAP Science Team

Four epochs of the history of the Universe

$H \equiv \frac{\dot{a}}{a}$ where $a(t)$ is a scale factor of an isotropic homogeneous spatially flat universe (a Friedmann-Lemaître-Robertson-Walker background):

$$ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2) + small\ perturbations$$

The history of the Universe in one line: four main epochs

$$? \longrightarrow DS \Longrightarrow FLRWRD \Longrightarrow FLRWMD \Longrightarrow \overline{DS} \longrightarrow ?$$

Geometry

$$|\dot{H}| << H^2 \Longrightarrow H = \frac{1}{2t} \Longrightarrow H = \frac{2}{3t} \Longrightarrow |\dot{H}| << H^2$$

Physics

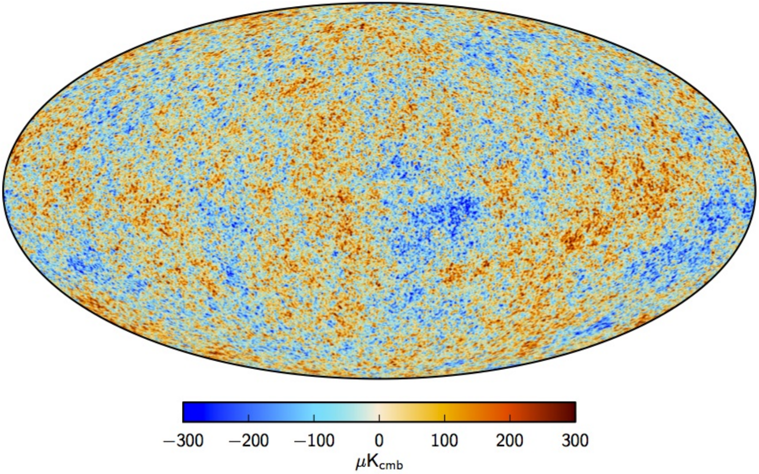
$$p \approx -\rho \Longrightarrow p = \rho/3 \Longrightarrow p \ll \rho \Longrightarrow p \approx -\rho$$

Duration in terms of the number of e-folds $\ln(a_{fin}/a_{in})$

$$> 60 \qquad \sim 55 \qquad 7.5 \qquad 0.5$$

CMB temperature anisotropy

Planck-2015: P. A. R. Ade et al., arXiv:1502.01589



Outcome of inflation

In the super-Hubble regime ($k \ll aH$) in the coordinate representation in the synchronous gauge with some additional conditions fixing it completely:

$$ds^2 = dt^2 - a^2(t)(\delta_{lm} + h_{lm})dx^l dx^m, \quad l, m = 1, 2, 3$$

$$h_{lm} = 2\xi(\mathbf{r})\delta_{lm} + \sum_{a=1}^{2} g^{(a)}(\mathbf{r})\, e_{lm}^{(a)}$$

$$e_l^{l(a)} = 0, \;\; g^{(a)}_{\;\;,l}\, e_m^{l(a)} = 0, \;\; e_{lm}^{(a)}\, e^{lm(a)} = 1$$

$\xi = -\mathcal{R}$ describes primordial scalar perturbations, g – primordial tensor perturbations (gravitational waves (GW)). The most important quantities:

$$P_\xi(k), \;\; \frac{d \ln P_\xi(k)}{d \ln k} \equiv n_s(k) - 1, \;\; r(k) \equiv \frac{P_g}{P_\xi}$$

Both $|n_s - 1|$ and $|r|$ are small during slow-roll inflation.

New cosmological parameters relevant to inflation

Now we have numbers: N. Agranim et al., arXiv:1807.06209

The primordial spectrum of scalar perturbations has been measured and its deviation from the flat spectrum $n_s = 1$ in the first order in $|n_s - 1| \sim N_H^{-1}$ has been discovered (using the multipole range $\ell > 40$):

$$< \mathcal{R}^2(\mathbf{r}) >= \int \frac{P_{\mathcal{R}}(k)}{k}\, dk, \ \ P_{\mathcal{R}}(k) = (2.10 \pm 0.03)\cdot 10^{-9} \left(\frac{k}{k_0}\right)^{n_s - 1}$$

$$k_0 = 0.05 \ \mathrm{Mpc}^{-1}, \ \ n_s - 1 = -0.035 \pm 0.004$$

Two fundamental observational constants of cosmology in addition to the three known ones (baryon-to-photon ratio, baryon-to-matter density and the cosmological constant). Existing inflationary models can predict (and predicted, in fact) one of them, namely $n_s - 1$, relating it finally to $N_H = \ln \frac{k_B T_\gamma}{\hbar H_0} \approx 67.2$. (note that $(1 - n_s)N_H \sim 2$).

The most recent upper limits on r

1. BICEP/Keck Collaboration: P. A. R. Ade et al., Phys. Rev. Lett. 127, 151301 (2021); arXiv:2110.00483:

$r_{0.05} < 0.036$ at the 95% C.L.

2. M. Tristram et al., Phys. Rev. D 105, 083524 (2022); arXiv:2112.07961:

$r_{0.05} < 0.032$ at the 95% C.L.

For comparison, in the chaotic inflationary model $V(\varphi) \propto |\varphi|^n$, $r = \frac{4n}{N}$, $1 - n_s = \frac{n+2}{2N}$. The r upper bound gives $n < 0.5$ for $N_{0.05} = (55 - 60)$, but then $1 - n_s \le 0.022$. Thus, this model is disfavoured by observational data.

The target prediction for r in the 3 simplest (one-parametric) inflationary models having $n_s - 1 = -\frac{2}{N}$ (the $R + R^2$, Higgs and combined Higgs-R^2 models) is

$$r = \frac{12}{N^2} = 3(n_s - 1)^2 \approx 0.004$$

Kinematic origin of scalar perturbations

Local duration of inflation in terms of $N_{tot} = \ln\left(\frac{a(t_{fin})}{a(t_{in})}\right)$ is different in different points of space: $N_{tot} = N_{tot}(\mathbf{r})$. Then ($\delta N$ formalism):

$$\xi(\mathbf{r}) = \delta N_{tot}(\mathbf{r})$$

Correct generalization to the non-linear case: the space-time metric after the end of inflation at super-Hubble scales

$$ds^2 = dt^2 - a^2(t)e^{2N_{tot}(\mathbf{r})}(dx^2 + dy^2 + dz^2)$$

First derived in A. A. Starobinsky, Phys. Lett. B 117, 175 (1982) in the case of one-field inflation.

Visualizing small differences in the number of e-folds

Duration of inflation in terms of e-folds was finite for all points inside our past light cone. For $\ell \lesssim 50$, neglecting the Silk and Doppler effects, as well as the ISW effect due the presence of dark energy,

$$\frac{\Delta T(\theta,\phi)}{T_\gamma} = -\frac{1}{5}\xi(r_{LSS},\theta,\phi) = -\frac{1}{5}\delta N_{tot}(r_{LSS},\theta,\phi)$$

For $n_s = 1, P_\xi = P_0$,

$$\ell(\ell+1)\,\langle(\Delta T/T_\gamma)^2_{lm}\rangle = \frac{2\pi}{25}P_0$$

For $\frac{\Delta T}{T} \sim 10^{-5}$, $\delta N \sim 5\times 10^{-5}$, and for $H \sim 10^{14}\,\mathrm{GeV}$, like in the minimal (one-parametric) inflationary models, $\delta t \sim 5t_{Pl}$!

Planck time intervals are seen by the naked eye!

Before inflation

Different possibilities were considered historically. The two simplest possibilities occurring in classical (possibly modified) gravity already.

1. Quasi-isotropic bounce of the scale factor with bounded curvature not exceeding that during inflation.
2. Generic anisotropic and inhomogeneous singularity with curvature much exceeding that during inflation.

A specific intermediate case: de Sitter 'Genesis': beginning from the exact contracting full de Sitter space-time at $t \to -\infty$ (AS, PLB 91, 99 (1980)).
Requires adding an additional term

$$R_i^l R_l^k - \frac{2}{3} R R_i^k - \frac{1}{2} \delta_i^k R_{lm} R^{lm} + \frac{1}{4} \delta_i^k R^2$$

to the rhs of the gravitational field equations. Not generic. May not be the 'ultimate' solution: a quantum system may not spend an infinite time in an unstable state.

Other more speculative possibilities

1. Creation of inflation "from nothing" (Grishchuk and Zeldovich, 1981).
One possibility among infinite number of others.

2. Our Universe was not an individual entity before inflationary stage, it was a part of some "Superuniverse" ("Multiverse" in modern terminology) (AS, Quantum Gravity, 1981).

3. More generally, any process may be responsible for the formation of inflationary stage in our Universe, that was called "creation from anything" in AS and Ya. B. Zeldovich, Sov. Sci. Rev. 1988.
Possible relation to de Sitter entropy.

Isotropic bounce with positive spatial curvature

Does not require modified gravity. Rather natural before inflationary stage since even a very small positive spatial curvature at present becomes important sufficiently early during inflation. The simplest model: closed FLRW universe filled by a massive scalar field (AS, Sov. Astron. Lett. 4, 82 (1978)). Also the 'slow roll' approximation presently used in all viable inflationary models was first introduced in this paper as 'slow climb' before bounce, $t_- < t < t_b$, and 'slow roll' after bounce at $t_b < t < t_+$.

$$\ln \frac{a}{a_\pm} = -\frac{m^2(t-t_\pm)^2}{6}, \ \phi = -\sqrt{\frac{2}{3}}\,\frac{m(t-t_\pm)}{\kappa}\, sgn\, H$$

$$\kappa^2 = 8\pi G, \ m|t-t_\pm| \gg 1, \ \kappa|\phi| \gg 1, \ ma_\pm \gg 1, \ ma\kappa|\phi| \gg 1$$

Generic, but probability of bounce and subsequent inflation is small for a large initial universe size $W \sim 1/(ma_-)$. Difficult to reach this from a low curvature state ("inflationary points").

Isotropic bounce with zero spatial curvature in scalar-tensor gravity

D. Polarski, A. A. Starobinsky, Y. Verbin. Bouncing cosmological isotropic solutions in scalar-tensor gravity. JCAP 2022, 052 (2022); arXiv:2111.07319.

In contrast to GR, scalar-tensor gravity admits breaking of weak and null energy conditions, so isotropic bounce is possible even in the absence of spatial curvature.

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2\kappa^2} + \frac{1}{2}\partial_\mu\Phi\partial^\mu\Phi - U(\Phi) - \frac{\xi}{2}R\Phi^2\right)$$

Bouncing solutions have been found for polynomial $U(\phi)$ negative in some range but bounded from below.

However, in all solutions either the Hubble function $H(t)$ becomes divergent at some finite moment of time before the bounce, or the effective gravitational constant $G_{eff} = G/(1 - \xi\kappa^2\Phi^2)$ becomes negative around the bounce.

As was shown in AS, Sov. Astron. Lett. 7, 36 (1981), in such solutions, arbitrarily small anisotropic perturbations diverge in the point there $G_{eff}^{-1} = 0$ and this results in the formation of generic anisotropic and inhomogeneous singularity at this moment preventing the transition to the region where G_{eff} is negative.

Bianchi-I type models in $f(R)$ gravity

Analytical and numerical investigation for $f(R) = R + R^2$ gravity in the Bianchi-I type model in D. Muller, A. Ricciardone, A. A. Starobinsky and A. V. Toporensky, Eur. Phys. J. C 78, 311 (2018). Two main types of singularities in $f(R)$ gravity with the same generic structure at $t \to 0$:

$$ds^2 = dt^2 - \sum_{i=1}^{3} |t|^{2p_i}\, a_l^{(i)} a_m^{(i)}\, dx^l dx^m,\ 0 < s \leq 3/2,\ u = s(2-s)$$

where $p_i < 1,\ s = \sum_i p_i,\ u = \sum_i p_i^2$ and $a_l^{(i)},\ p_i$ are functions of $\mathbf{r}$. Here $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$.

Type A. $1 \leq s \leq 3/2,\ |f'(R)| \propto |t|^{1-s} \to +\infty\,.$
Type B. $0 < s < 1,\ R \to R_0 < 0,\ f'(R_0) = 0\,.$
In addition, there can exist an isotropic Big Rip type singularity with $s = -3(n-1)(2n-1)/(n-2),\ u = s^2/3$ in the future evolution if $f(R) \propto R^n,\ n > 2$ for $R \to \infty$.

Bianchi-I type models with inflation in R^2 gravity

For $f(R) = R^2$, even an exact solution can be found.

$$ds^2 = \tanh^{2\alpha}\left(\frac{3H_0 t}{2}\right)\left(dt^2 - \sum_{i=1}^{3} a_i^2(t)dx_i^2\right)$$

$$a_i(t) = \sinh^{1/3}(3H_0 t)\tanh^{\beta_i}\left(\frac{3H_0 t}{2}\right), \ \sum_i \beta_i = 0, \ \sum_i \beta_i^2 < \frac{2}{3}$$

$$\alpha^2 = \frac{\frac{2}{3} - \sum_i \beta_i^2}{6}, \quad \alpha > 0$$

Next step: relate arbitrary functions of spatial coordinates in the generic solution near a curvature singularity to those in the quasi-de Sitter solution. Spatial gradients may become important for some period before the beginning of inflation.

The same structure of generic singularity for a non-minimally coupled scalar field (scalar-tensor gravity)

$$S = \int \left(f(\phi)R + \frac{1}{2}\phi_{,\mu}\phi^{,\mu} - V(\phi) \right) \sqrt{-g}\, d^4x + S_m$$

$$f(\phi) = \frac{1}{2\kappa^2} - \xi\phi^2$$

Type A. $\xi < 0, |\phi| \to \infty$.
Type B. $\xi > 0, |\phi| \to 1/\sqrt{2\xi}\kappa$.
The asymptotic regimes and a number of exact solutions in the Bianchi type I model are presented in A. Yu. Kamenshchik, E. O. Pozdeeva, A. A. Starobinsky, A. Tronconi, G. Venturi and S. Yu. Vernov, Phys. Rev. D 97, 023536 (2018) with some of them borrowed from A. A. Starobinsky, MS Degree thesis, Moscow State University, 1972, unpublished.
Anisotropy grows towards singularity generically like in GR.

Horndeski gravity

Generalization of scalar-tensor gravity in the scalar sector without introducing new (scalar) degrees of freedom.

$$\mathcal{S} = \int (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5) \sqrt{-g}\, d^4x \,,$$

$$\mathcal{L}_2 = G_2(\phi, X), \quad \mathcal{L}_3 = -G_3(\phi, X)\, \Box\phi \,,$$

$$\mathcal{L}_4 = G_4(\phi, X)R + G_{4X}(\phi, X) \left[(\Box\phi)^2 - (\nabla_\mu \nabla_\nu \phi)^2 \right] \,,$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu}\, \nabla^\mu \nabla^\nu \phi - \frac{1}{6} G_{5X}(\phi, X) \times$$

$$\times \left[(\Box\phi)^3 - 3\Box\phi(\nabla_\mu \nabla_\nu \phi)^2 + 2\, (\nabla_\mu \nabla_\nu \phi)^3 \right] .$$

Here $X = -\frac{1}{2}\nabla^\mu\phi\nabla_\mu\phi$, $G_{AX} \equiv \partial G_A/\partial X$. R and $G_{\mu\nu}$ are the Ricci scalar and the Einstein tensor, $R = -G^\mu_\mu$.

Specific types of Horndeski gravity

1. $G_2 = X - V(\phi),\ \ G_3 = G_5 = 0,\ \ G_4 = const$ - GR + a self-interacting scalar field minimally coupled to gravity.
2. $G_2 = X - V(\phi),\ \ G_3 = G_5 = 0,\ \ G_4 = G_4(\phi)$ - scalar-tensor gravity.
3. $G_2 = G_2(\phi, X),\ \ G_3 = G_5 = 0,\ \ G_4 = const$ - K-essence theory.
4. $G_3(\phi, X) \neq 0$ - Kinetic Gravity Braiding (KGB) theory.

The theory with $G_4 = G_4(\phi),\ \ G_5 = 0$ is the most general Horndeski model in which the sound speed of tensor perturbations is exactly equal to the speed of light in the presence of the scalar field ϕ. However, we will not restrict ourselves to this constraint.

Bianchi-I type solutions of Horndeski gravity

R. Galeev, R. Muharlyamov, A. A. Starobinsky, S. V. Sushkov and M. S. Volkov, Phys. Rev. D 103, 104015 (2021); arXiv:2102.10981.

The Bianchi-I type metric:

$$ds^2 = -dt^2 + \mathrm{a}_1^2 \, dx_1^2 + \mathrm{a}_2^2 \, dx_2^2 + \mathrm{a}_3^2 \, dx_3^2 \,.$$

Parametrization of the three scale factors:

$$\mathrm{a}_1 = \mathrm{a}\, e^{\beta_+ + \sqrt{3}\beta_-}, \;\; \mathrm{a}_2 = \mathrm{a}\, e^{\beta_+ - \sqrt{3}\beta_-}, \;\; \mathrm{a}_3 = \mathrm{a}\, e^{-2\beta_+} \,.$$

Let us denote

$$\mathcal{G} = 2G_4 - 2G_{4X}\dot{\phi}^2 + G_{5\phi}\dot{\phi}^2 \,, \;\; \sigma^2 = \dot{\beta}_+^2 + \dot{\beta}_-^2 \,, \quad H = \frac{\dot{\mathrm{a}}}{\mathrm{a}} \,,$$

$$H_1 = H + \dot{\beta}_+ + \sqrt{3}\dot{\beta}_- \,, \; H_2 = H + \dot{\beta}_+ - \sqrt{3}\dot{\beta}_- \,, \; H_3 = H - 2\dot{\beta}_+ \,.$$

4 equations for 4 variables

The field equation:

$$\frac{1}{\mathrm{a}^3}\frac{d}{dt}(\mathrm{a}^3\mathcal{J}) = \mathcal{P}\,,$$

$$\mathcal{J} = \dot{\phi}\Big[G_{2X} - 2G_{3\phi} + 3H\dot{\phi}(G_{3X} - 2G_{4X\phi}) + + G_0^0\,(-2G_{4X} -$$

$$-2\dot{\phi}^2 G_{4XX} + 2G_{5\phi} + G_{5X\phi}\dot{\phi}^2) + H_1 H_2 H_3(3G_{5X}\dot{\phi} + G_{5XX}\dot{\phi}^3)\Big]\,,$$

$$\mathcal{P} = G_{2\phi} - \dot{\phi}^2(G_{3\phi\phi} + G_{3X\phi}\ddot{\phi}) + RG_{4\phi} + 2G_{4X\phi}\dot{\phi}(3\ddot{\phi}H - \dot{\phi}G_0^0) +$$

$$+ G_0^0 G_{5\phi\phi}\dot{\phi}^2 + G_{5X\phi}\dot{\phi}^3 H_1 H_2 H_3\,.$$

The first order G_0^0 equation:

$$3\left(H^2-\sigma^2\right)\left(\mathcal{G}-2G_{4X}\dot{\phi}^2-2G_{4XX}\dot{\phi}^4+2G_{5\phi}\dot{\phi}^2+G_{5X\phi}\dot{\phi}^4\right)=$$

$$=-G_2+\dot{\phi}^2G_{2X}+3G_{3X}H\dot{\phi}^3-G_{3\phi}\dot{\phi}^2-6G_{4\phi}H\dot{\phi}-6G_{4X\phi}H\dot{\phi}^3+$$

$$+\dot{\phi}^3(5G_{5X}+G_{5XX}\dot{\phi}^2)(H-2\dot{\beta}_+)\left[(H+\dot{\beta}_+)^2-3\dot{\beta}_-^2\right].$$

Two first order equations for anisotropy factors:

$$\mathcal{G}\dot{\beta}_+ + G_{5X}\dot{\phi}^3\left(\dot{\beta}_-^2-\dot{\beta}_+^2-H\dot{\beta}_+\right)=\frac{C_+}{\mathrm{a}^3},$$

$$\mathcal{G}\dot{\beta}_- + G_{5X}\dot{\phi}^3\left(2\dot{\beta}_+\dot{\beta}_- - H\dot{\beta}_-\right)=\frac{C_-}{\mathrm{a}^3}.$$

Behaviour of anisotropy towards curvature singularity

Anisotropy grows towards singularity like in GR in models where $G_4 = G_4(\phi)$, $G_5 = 0$. However, if $G_4 = G_4(X)$, or $G_5 = G_5(X)$, it grows first but then decrease. This effect was first found earlier in A. A. Starobinsky, S. V. Sushkov and M. S. Volkov, Phys. Rev. D 101, 064039 (2020); arXiv:1912.12320 for the specific case $G_5 = const$. Such models show gradient instabilities at early times (c_s^2, or c_t^2, or both become negative). Thus, their initial phase, although not anisotropic, cannot be isotropic either. It should therefore be inhomogeneous. At the same time, it is possible that a systematic analysis of theories with more general $G_4(\phi, X)$ and/or $G_5(\phi, X)$ may reveal models free of instabilities.

Conclusions

- Duration of inflationary stage inside our past light-cone was finite. Moreover, we directly see the difference of this duration in terms of the number of its e-folds between different points at the LSS from the large-scale CMB temperature anisotropy.
- In contrast to the inflationary stage itself, at present we have no reliable and unambiguous observational data regarding previous history of our Universe before inflation.
- Among many possible variants of the pre-inflationary history, the two simplest and most conservative alternatives are quasi-isotropic bounce due to positive spatial curvature in some finite region of space, or generic anisotropic singularity with curvature much exceeding that during the observable part of inflation.

- With a positive spatial curvature, generic bounce of an isotropic homogeneous universe is possible. Moreover, the assumption of homogeneity can be omitted: it is sufficient that spatial curvature is positive in some finite region of space only. Then only a small part of a collapsing universe ("inflationary points") bounces and inflate after that. Asymmetry between past and future due to global inhomogeneity as a result of this (a kind of "time arrow").
- In scalar-tensor gravity with $U(\phi)$ negative in some range but bounded from below, isotropic bouncing solutions with a finite Hubble function $H(t)$ are possible even in the absence of spatial curvature, but they require G_{eff} becoming negative around the bounce and, thus, are unstable with respect to the formation of generic and inhomogeneous curvature singularity at the moment when $G_{eff}^{-1} = 0$. Thus, they are unphysical.

- In scalar-tensor or $f(R)$ gravity, generic asymptotic behaviour near singularity is quasi-regular (finite number of BLH oscillations) with $R^2 \ll R_{\alpha\beta}R^{\alpha\beta}$.
- In Horndeski gravity, anisotropy behaviour near singularity may be completely different from that in GR for models in which the GW velocity in presence of the scalar field is different from the light one, though this difference may be arbitrarily small at present. Moreover, even the structure of the curvature singularity itself may be different.